

BARYON AND TIME ASYMMETRIES OF THE UNIVERSE

Andro BARNAVELI¹ and Merab GOGBERASHVILI²

*Institute of Physics of the Georgian Academy of Sciences,
Tamarashvili str. 6, Tbilisi 380077, Republic of Georgia.*

Abstract

This paper is devoted to the investigation of connection between two apparent asymmetries of the nature — time-asymmetry and Baryon Asymmetry of the Universe (BAU). The brief review of this subjects is given. We consider the particle behavior in curved space-time and the possibility of T - and CPT -violation by the universe expansion. If these symmetries are violated we can dispense with the nonequilibrium condition which is usually considered as the one of necessary ingredients for BAU-generation. Such mechanism of GUT-scale baryogenesis can provide the observed value of baryon asymmetry. We show this on the example of minimal $SU(5)$ model which usually fails to explain the observed BAU without taking into account gravitational effects. Predominance of matter over antimatter and the cosmological arrow of time (the time-direction in which the Universe expands) seem to be connected facts and, possibly, BAU is the one of observable facts of CPT -violation in nature.

¹e-mail: bart@physics.iberiapac.ge

²e-mail: gogber@physics.iberiapac.ge

Contents

1	Introduction	5
2	Particles in the expanding Universe	9
2.1	The model of expanding Universe	10
2.2	Fields in conformally equivalent spaces	13
2.3	Quantum fields in FRW space	15
2.4	Short-time-interval approximation	19
3	Effects of CPT-violation in FRW Universe	23
3.1	CPT -theorem in Minkowski space	23
3.2	The direct- and inverse-decay rate difference	26
3.3	Particle-antiparticle mass-difference	28
4	Cosmological arrow of time and BAU	31
4.1	The problem of baryon asymmetry	31
4.2	Particle-antiparticle mass-difference and Baryogenesis in $SU(5)$ model	35
4.3	Calculation of BAU caused by the Universe expansion	40
	Bibliography	42

Chapter 1

Introduction

For a long time much attention was paid to the existence of different asymmetries of nature. For example to the apparent asymmetry between matter and antimatter and to existence of preferred time direction in the Universe. Here we want to discuss the connections between these asymmetries.

A striking feature of our everyday experience about time refers to its immutable flow from past to present and from present to future. Remarkably enough the reality of time irreversibility has been a point of recurrent polemics in physics since the debate between Boltzmann and Zermelo late last century (for details see [1]). The question of "time arrow" was discussed quite widely (see e.g. [1, 2, 3, 4, 5]). On the matter of irreversibility, Einstein wrote in a letter to his friend Michele Besso: "There is no irreversibility in the basic laws of physics. You have to accept the idea that subjective time with its emphasis on the now has no objective meaning". Indeed the local physical laws which are well-known and which people understand are all symmetric with respect to time (it would be more correct to say that they do not change after symmetry transformations C , P and T , where C interchanges particle with its antiparticle, P — the parity transformation changes right with left, T — changes the time direction to opposite), but nevertheless at the macroscopic level there is an apparent time-asymmetry.

One can point out some different (at first sight) time arrows (see e.g. [5]):

- a). Thermodynamical arrow of time, showing the time direction in which the entropy (or disorder) does increase;
- b). Psychological arrow of time. This is the direction of our "feeling" of time — the direction at which we remember our past but not future;
- c). The cosmological arrow of time — the time direction in which the Universe expands but not collapses.

To this three cases one can add also:

— The decay of K^0 -meson (see also [6, 7]), where the T -violating component is only the 10^{-9} -th part of T -conserving component; here T -violation is derived from CP -violation ($\sim 10^{-9}$) under the assumption that CPT -violation, if such exists, is sufficiently smaller ($\ll 10^{-9}$);

— The quantum-mechanical observations. The procedure of observation in quantum mechanics seems to be connected with some non-reversible process and assumes the entropy increase;

— The delay of radiation. Any perturbation is expanding away from its source in

all directions and reaches some distant point after particular time-interval. The reversed picture, when perturbations are gathering from the whole space to the source-point, seemingly, never does occur;

— Correlation of black and white holes. General Relativity is a time-symmetric theory. Thus to every time-asymmetric process, such as the formation of black holes, must correspond the other process with the opposite time-behavior, such as would be formation of white holes. There are different opinions about existence of white holes (see e.g. [8]), but the most serious considerations lead to the conclusion that, seemingly, such objects do not exist [5].

One can try to find some relations between these arrows. For example the delay of radiation can be explained in terms of entropy increase i.e. by the existence of thermodynamical arrow of time [5]. Besides, basing on the statement that the Universe has no boundaries and using the weak anthropic principle ("we live in such place in the Universe and at that stage of its evolution, where our life is possible") [9] it is possible to show [4] that the thermodynamical, psychological and cosmological arrows of time have the same direction.

The psychological arrow of time is determined by thermodynamical one and their directions coincide. One can show it on the example of computer. When one places some information in computer's memory, he has to expend some energy which consequently transforms into heat and thus increases the entropy of the Universe (this growth of entropy is much greater than its decrease which will occur since after the process of remembering the memory of computer from the state of disorder transfers to the state of order). Thus the time-direction in which computer does remember is the direction in which the entropy does increase. "The entropy increases with time because we measure the time in the direction in which entropy increases" [4].

If one assumes that the Universe has no boundaries, then the well-defined thermodynamical and cosmological arrows of time must exist, however their directions have not to coincide inevitably. But only in the case of coincidence of their directions the conditions for existence of human beings could appear. Certainly, the condition of boundary absence means that the history of the Universe is finite but it has no boundaries, no edges and no singularities [4]. Then the initial moment of time-count must be the regular point of space-time and thus the Universe began its expansion from quite isotropic and ordered state. This state could have some fluctuations of particle density and velocities, but they should be sufficiently small to satisfy the uncertainty principle. As the Universe expanded these fluctuations increased and the Universe transferred from the isotropic and ordered state into the state which is unisotropic and disordered. This can explain the existence of thermodynamical arrow of time. If the Universe finishes its expansion and begins to collapse then disorder will continue to grow and thus the thermodynamical and psychological arrows of time will not change their direction in collapsing Universe. In this case thermodynamical and cosmological arrows of time will have the opposite directions. However here works the weak anthropic principle — conditions at the stage of the Universe collapse will not be valid for the existence of human beings [4]. Thus the human life is possible only at the stage of expansion and in this case the directions of cosmological, thermodynamical and psychological arrows of time coincide.

All these arrows of time indicate that the nature does not require exact symmetry

in time. One physical consequence of this asymmetry is the decay of K^0 -meson. The other physical process, which can be explained by the time asymmetry is the preferential creation of baryons over antibaryons in the early Universe. The consequence of this process is the predominance of matter over antimatter known as a Baryon Asymmetry of the Universe (BAU).

BAU is the other observational asymmetry in cosmology. We all know there are no antimatter bodies in the solar system. Solar cosmic rays are evidence that the sun is also composed of matter. Beyond the solar system cosmic rays prove that the asymmetry of matter and antimatter extends (at least) throughout our galaxy. Certainly the cosmic rays contain a 10^4 times less amount of antiprotons than protons that is consistent with the assumption that the antiprotons are secondaries; the flux of anti-nuclei is less than 10^{-5} that of nuclei; and there is no clear detection of an antinucleus. The clusters of galaxies either consist only of matter, because otherwise we should observe x - and γ -ray emission from matter-antimatter annihilations between the galaxies and intergalaxy gas. There is too small information on scales larger than clusters of galaxies. For a review see [10, 11, 12].

We shall show that expansion of the Universe — cosmological arrow of time — breaks Poincare-invariance and causes violation of T - and CPT -symmetries. This fact can be expressed in such phenomena as the different rates of direct and inverse decays of particles or mass difference between particles and antiparticles. As a consequence of the latter the particles in the early Universe could be created more preferentially than antiparticles and this had caused generation of Baryon Asymmetry of the Universe (BAU). Thus BAU and cosmological time-arrow are connected phenomena. Moreover, the sign of BAU and the direction of cosmological arrow of time are strongly correlated. We will see that in collapsing Universe the predominance of antimatter will occur.

In section 2 we discuss the problems of particle theory in curved space-time, in particular, in Friedmann-Robertson-Walker (FRW) space. In subsection 2.1 we review briefly the standard cosmological model of expanding Universe. FRW metric is conformally equivalent to Minkowski metric and so in subsection 2.2 the formulae of conformal transformations are given. In subsections 2.3 and 2.4 the problems of quantization and corpuscular interpretation of quantum fields are considered on the example of scalar field.

The section 3 is dedicated to investigation of possible violation of CPT -theorem in FRW spaces. In subsection 3.1 the CPT -theorem in Minkowski space is recalled briefly. Then we consider the effects of CPT -violation caused by Universe expansion, such as the difference between the direct and inverse particle-decay rates (subsection 3.2) and the mass difference between particles and antiparticles (subsection 3.3).

In the last section 4 the connection between time and baryon asymmetries of the Universe is discussed. In subsection 4.1 we review the problem of BAU, in subsection 4.2 the calculation of BAU in the frames of $SU(5)$ GUT-model is given under the assumption that T and CPT is violated and the nonequilibrium condition of baryogenesis is not necessary. In subsection 4.3 we show that such model of baryogenesis really can provide the observed value of BAU for the different regimes of the Universe expansion.

Chapter 2

Particles in the expanding Universe

The interaction of particles with gravitational fields usually is neglected with reference to smallness of gravitational constant G . In the early Universe however gravitational fields have been so strong that particle interactions generally must not be described by quantum field theory in Minkowski space (the same logic has to be applied to particle interactions in the vicinity of strongly gravitating objects such as black holes, cosmic strings, monopoles or domain walls).

Gravitational effects in the particle physics have been considered in a number of papers. Here we shall refer to [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. Also see books [26, 27] and many other references therein.

As a whole, introducing the interaction of quantum fields with gravity is one of the principle questions of the Field Theory, since we have not yet the successful quantum theory of gravitation. The difficulty appears mainly due to nonlinear structure of Einstein equations and to the fact that gravitational theory is invariant under the infinite-parameter group of general coordinate transformations.

The simplest possible approach to investigate the influence of strong gravitational fields upon particle transmutations is the theory of quantized elementary particle fields propagating in a classical curved background space-time.

The well-known problem of investigations of matter fields in curved space-time is the problem of interpretation of quantum fields in terms of particles and of the description of vacuum state. Particles can be interpreted as the energy quanta. The measured energy corresponds to the eigenstate of Hamiltonian which must have the diagonalized form. Particles and antiparticles are associated with those creation-annihilation operators, which diagonalize the Hamiltonian.

In Minkowski space the corpuscular interpretation of free fields is based on the invariance with respect to Poincare group. We can introduce time-conserved separation of field operators into positive and negative frequency parts $\psi^\pm(x)$ and describe vacuum state $|0\rangle$ as

$$\psi^-(x)|0\rangle = 0.$$

This modes are connected with the natural orthogonal coordinate system (t, x, y, z) which is associated with Poincare group. Vector $\partial/\partial t$ is the Killing vector of Minkowski space orthogonal to spacelike hypersurfaces $t = \text{const}$, while modes ψ^\pm are eigenfunc-

tions of this vector with eigenvalues $\mp i\omega$:

$$\frac{\partial \psi^\pm(x)}{\partial t} = \mp i\omega \psi^\pm.$$

Such construction of Fock space does not depend on the choice of the basis in the space of classical solutions of field equations.

In general case of Riemannian spaces one has no such principle to choose the basic functions and describe the vacuum. The Poincare group is not a symmetry group of Riemannian space and so the Killing vectors, by means of which one can define the positive- and negative-frequency solutions, do not exist. The mixing of positive and negative frequencies takes place. Already for the case of free fields which are almost trivial in the flat space this can be exhibited by particle creation from vacuum. Moreover, if matter fields are mutually interacting in presence of a strong gravitational field, new effects can arise, such as particle production in some interactions and violation of *CPT*-invariance which can be exhibited in different rates of direct and inverse decays of particles or in the mass difference between particles and antiparticles.

In this chapter we shall discuss briefly some aspects of field theory in curved space-time which are necessary to understand the connection between the Universe expansion and BAU.

2.1 The model of expanding Universe

Before investigating the influence of the Universe expansion on particle interactions let us first describe the simplest big-bang model — the Friedmann-Robertson-Walker (FRW) Universe (see e.g. [28]).

We shall perform our calculation in the background of curved space-time metric. The gravitational field is identified with the space-time metric $g_{\mu\nu}$, dynamics of which is described by Einstein-Hilbert action

$$S_g = -\frac{1}{16\pi G} \cdot \int \sqrt{-g}(R + 2\Lambda)dV,$$

where, for the sake of generality, we have included a cosmological constant Λ . Here $G \sim 1/M_{Pl}^2$ is the Newton's constant ($M_{Pl} \sim 10^{19} GeV$ is the Planck mass), R is the scalar curvature, g is the determinant of metric and we have chosen the signature $(+, -, -, -)$. The matter is generically described by a matter action S_m which provides a source for gravitational interaction, i.e. the right-hand side of Einstein equations

$$R_{\mu\nu} - \frac{1}{2}(R + 2\Lambda)g_{\mu\nu} = 8\pi GT_{\mu\nu},$$

where

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \cdot \frac{\delta S_m}{\delta g^{\mu\nu}},$$

is the energy-momentum tensor.

Under assumptions of homogeneity and isotropy the metric of the universe can be written in the Friedmann-Robertson-Walker form:

$$ds^2 = dt^2 - a^2(t)dl^2, \quad (2.1)$$

where t is proper time measured by a comoving observer, $a(t)$ is the scale factor and

$$dl^2 = \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

is the interval of 3-space of constant curvature. Here r , θ and φ are comoving spherical coordinates. The constant k is a measure of curvature and its values $k = -1$, 1 or 0 correspond to open, closed or flat isotropic models. It is easy to see that if we introduce a new variable

$$\bar{r} = \frac{1}{\sqrt{k}} \arcsin(\sqrt{k}r)$$

then dl^2 takes the form

$$dl^2 = d\bar{r}^2 + f^2(\bar{r})(d\theta^2 + \sin^2\theta d\varphi^2),$$

where $f^2(\bar{r})$ is $\text{sh}^2\bar{r}$ for the open Universe ($k = -1$), $\sin^2\bar{r}$ for the closed Universe ($k = 1$) and \bar{r}^2 for the flat Universe ($k = 0$).

To solve the Einstein equations and study the matter fields in the background of FRW metric (2.1) it is convenient to introduce the dimensionless "conformal time" variable τ by the relation

$$\tau = \int \frac{dt}{a(t)}. \quad (2.2)$$

Then the interval (2.1) takes the form conformal to the flat one:

$$ds^2 = a^2(\tau)(d\tau^2 - dl^2), \quad (2.3)$$

i.e. for any value of k the (2.3) is conformally equivalent to Minkowski metric.

The curvature tensor for metric (2.3) has the form:

$$\begin{aligned} R_0^0 &= \frac{3}{a^4}(\dot{a}^2 - a\ddot{a}); \\ R_j^i &= -\frac{1}{a^4}(2ka^2 + \dot{a}^2 + a\ddot{a})\delta_j^i; \\ R_i^0 &= 0; \quad i, j = 1, 2, 3, \end{aligned}$$

where the overdot denotes derivative with respect to τ .

For the solution of the Einstein equations the energy-momentum tensor of matter and cosmological constant must be determined also. Usually it is taken the perfect-fluid ansatz for the gravity source:

$$\begin{aligned} T_0^0 &= \rho; \\ T_j^i &= -\delta_j^i P, \end{aligned}$$

where ρ is the total energy density of the matter and P is the isotropic pressure, and the vanishing cosmological constant Λ is assumed. Then for metric (2.3) the Einstein equations are:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \quad (2.4)$$

$$\dot{\rho} + 3H(\rho + P) = 0. \quad (2.5)$$

Here the Hubble parameter H characterizes the intensity of gravitational field and is determined by the scale factor a in the following way:

$$H = \frac{1}{a} \frac{da}{dt} = \frac{\dot{a}}{a}.$$

On the present stage of the Universe evolution

$$H \simeq 50 \div 100 \frac{km}{sec \cdot Mps} \sim 10^{-26} cm^{-1}.$$

It is easy to find from (2.4) and (2.5) that

$$\begin{aligned} \tau &= \pm \int \frac{da}{a\sqrt{K^2\rho a^2 - k}}; \\ 3\ln a &= - \int \frac{d\rho}{(\rho + P)} + const, \end{aligned} \quad (2.6)$$

where

$$K \equiv \sqrt{\frac{8\pi G}{3}}. \quad (2.7)$$

The system (2.4) and (2.5) contains three unknown functions — P , ρ , a . In order to get the unique solution of (2.4) and (2.5) (up to integration constants) we have to provide an equation of state, usually written as a fixed relation between ρ and P . For instance, $\rho = -P$ corresponds to vacuum dominance era ($a \sim e^{Ht}$, $H = K\sqrt{\rho}$); $P = \rho/3$ gives the radiation-dominated cosmology ($a \sim \sqrt{t}$), while $P = 0$ gives the matter-dominated ($a \sim t^{2/3}$) expansion of the present era.

In scenarios of inflation the main ingredient is a scalar field ϕ , the so-called inflaton, whose potential $V(\phi)$ provides energy and pressure during an early era of vacuum dominance. Since the Lorentz-invariance of vacuum gives that $T_{\mu\nu} \sim g_{\mu\nu}$, the inflation provides a negative pressure equal in magnitude to the energy density giving an accelerated expansion. While, since the usual matter has a non-negative pressure, the radiation- and matter-dominated models are always decelerating ($\ddot{a} < 0$).

Here we would like to consider the matter fields at the energies of Grand Unification $M_{GUT} \sim 10^{15} \div 10^{16} GeV$. This corresponds to the age of the Universe of the order of

$$t \sim \frac{1}{M_{GUT}},$$

when inflation or radiation-dominated Universe is considered usually. At this stage of evolution the second term at the right side of the equation (2.4) can be neglected

and one can use the flat FRW model. Now on the base of (2.2) and (2.6) it is easy to write the relation between time and scale factor:

$$t = \int_0^\tau a(\tau) d\tau = \frac{1}{K} \int_{a_0}^a \frac{da}{a\sqrt{\rho}}.$$

Here the initial moments t_0 and τ_0 , corresponding to GUT energy, are taken to be zero for the convenience and a_0 is the initial value of the scale factor.

For the vacuum-dominated model ($\rho = \text{const}$) equations (2.6) show that the Universe is described by de Sitter expansion law:

$$a(t) = a_0 e^{Ht} \quad \text{or} \quad a(\tau) = \frac{a_0}{1 - a_0 H \tau} \quad (2.8)$$

with the Hubble parameter $H = K\sqrt{\rho}$.

If one considers the radiation-dominated stage ($\rho \sim a^{-4}$), then according to (2.6) the scale factor depends on time as

$$a(t) = \sqrt{a_0^2 + 2Kt} \quad \text{or} \quad a(\tau) = a_0 + K\tau. \quad (2.9)$$

2.2 Fields in conformally equivalent spaces

As we have seen in previous section, the FRW space is conformally equivalent to Minkowski space:

$$\bar{g}_{\mu\nu} = a^2(\tau) \eta_{\mu\nu}.$$

This feature simplifies greatly the calculations in FRW spaces. Thus it would be worth reminding the formalism of conformal transformations.

There exist two types of transformations named as conformal. The conformal transformations of coordinates $x_\mu \rightarrow x'_\mu$ change only the meanings of the points on some card and do not change the geometry.

In the field theory the more useful are the conformal transformations of metric:

$$g_{\mu\nu} \rightarrow \bar{g}_{\mu\nu} = \Omega^2(x) \cdot g_{\mu\nu}, \quad (2.10)$$

which, by contrast to coordinate transformations, contract or stretches the manifold. In (2.10) the $\Omega(x)$ is some continuous, nonvanishing, finite and real function. The conformal transformations are used, for example, in Penrose conformal diagrams, where the whole space-time is stretched on compact manifold. The other example is the scale-covariant theory of gravitation which provides an interesting alternative for conventional Einstein theory [29].

The conformal transformation (2.10) changes Christoffel symbols, Ricchi tensor and the scalar curvature in the following way

$$\begin{aligned} \Gamma_{\mu\nu}^\rho &\rightarrow \bar{\Gamma}_{\mu\nu}^\rho = \Gamma_{\mu\nu}^\rho + \frac{1}{\Omega} \left(\delta_\mu^\rho D_\nu \Omega + \delta_\nu^\rho D_\mu \Omega - g_{\mu\nu} g^{\rho\alpha} D_\alpha \Omega \right), \\ R_\mu^\nu &\rightarrow \bar{R}_\mu^\nu = \frac{1}{\Omega^2} R_\mu^\nu - \frac{2}{\Omega} g^{\rho\nu} D_\mu D_\rho \left(\frac{1}{\Omega} \right) + \frac{1}{2\Omega^4} \delta_\mu^\nu g^{\rho\sigma} D_\rho D_\sigma \left(\frac{1}{\Omega^2} \right), \\ R &\rightarrow \bar{R} = \frac{1}{\Omega^2} R + \frac{4}{\Omega^3} g^{\mu\nu} D_\mu D_\nu \Omega, \end{aligned} \quad (2.11)$$

where D_μ is the covariant derivative with respect to initial metric $g_{\mu\nu}$.

From (2.10) and (2.11) it is easy to derive the following usefull transformation for the scalar field:

$$\left(\nabla^2 + \frac{1}{6}R\right)\varphi \rightarrow \left(\bar{\nabla}^2 + \frac{1}{6}\bar{R}\right)\bar{\varphi} = \frac{1}{\Omega^3} \left(\nabla^2 + \frac{1}{6}R\right)\varphi, \quad (2.12)$$

where

$$\nabla^2 = \frac{1}{\sqrt{-g}} \cdot \partial \left[\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi \right]$$

and

$$\varphi \rightarrow \bar{\varphi} = \frac{1}{\Omega} \varphi. \quad (2.13)$$

The conformal invariance plays the essential role in determination of matter field coupling with gravitation. Indeed, massless particles are not characterized by definite Compton wavelength $\lambda_c = 1/m$ and must have the same behavior in conformally equivalent Riemannian spaces. It means that their motion equations must be invariant under conformal transformations (2.10).

Under this assumption one has to write the equation for massless scalar field φ in 4-dimensional Riemannian space as

$$\left(\nabla^2 + \frac{1}{6}R\right)\varphi = 0. \quad (2.14)$$

Then this equation is conformally invariant. Certainly, from (2.12) and (2.13) it is easy to see that from (2.14) immediately follows

$$\left(\bar{\nabla}^2 + \frac{1}{6}\bar{R}\right)\bar{\varphi} = 0.$$

Equation for the massless vector field A_μ

$$D^\nu F_{\mu\nu} = 0,$$

where

$$F_{\mu\nu} = D_\nu A_\mu - D_\mu A_\nu = \partial_\nu A_\mu - \partial_\mu A_\nu,$$

in four dimensional space is conformally invariant without any requirement for variation of A_μ under conformal transformations since $F_{\mu\nu}$ does not change its form in curved spaces, in particular under conformal transformations (2.10).

Dirac equation for the massless spinor field ψ has the form:

$$i\gamma^\mu D_\mu \psi = 0,$$

where

$$\gamma^\mu = h_a^\mu \gamma^a$$

and h_a^μ is a tetrad gravitational field. This equation is conformally invariant if we require that under transformation (2.10) the field ψ varies in the following way:

$$\psi \rightarrow \bar{\psi} = \frac{1}{\Omega^{3/2}} \psi.$$

The fact that FRW metric and Minkowski metric are conformally equivalent gives us the opportunity to write the fields in cosmological spaces by using their form in Minkowski space. For example, for the scalar fields the account of the space-time curvature is equal to the transformation of field

$$\varphi \rightarrow a(\tau) \cdot \varphi$$

and of the Hamiltonian [26]

$$\mathcal{H}_\varphi^0 \rightarrow \mathcal{H}_\varphi = a^2(\tau) \cdot \mathcal{H}_\varphi^0, \quad (2.15)$$

where \mathcal{H}^0 is Hamiltonian in Minkowski space.

The vector fields are conformally invariant and their Hamiltonian does not change its form in FRW space.

For the spinor fields in FRW space-time we have

$$\psi \rightarrow a^{3/2}(\tau)\psi$$

and for their Hamiltonian [26]

$$\mathcal{H}_\psi = a^3(\tau)\mathcal{H}_\psi^0. \quad (2.16)$$

2.3 Quantum fields in FRW space

For a wide class of models gravitational field can be considered as an external classical field, because the quantum nature of gravity is essential only for strong gravitational fields when the space-time curvature is characterized by Planck length

$$l_{Pl} = \frac{1}{M_{Pl}} = \sqrt{G} \sim 1.6 \cdot 10^{-33} \text{cm},$$

while for the matter fields quantum effects appear at the Compton-length distances

$$l_C = \frac{1}{m},$$

where m is the particle mass.

Thus on the GUT scale, which we are considering in this paper, ($m \sim 10^{15 \div 16} \text{GeV} \ll M_{Pl} \sim 10^{19} \text{GeV}$) gravity can be considered as a classical field and one can investigate matter fields in the background of curved space-time. As an example let us consider quantization of a scalar (pseudoscalar) field φ . For the other-type fields the problems arising in the FRW spaces are the similar.

For the comparison let us recall that the scalar field $\varphi(t, \vec{x}) \equiv \varphi(x_\mu)$ in Minkowski space-time is described by the action

$$S = \int \frac{1}{2} \eta^{\mu\nu} (\partial_\mu \varphi \partial_\nu \varphi - m^2 \varphi^2) d^4x,$$

where $\eta^{\mu\nu} = (1, -1, -1, -1)$ is the metric of Minkowski space. It satisfies the motion equation

$$(\eta^{\mu\nu} \partial_\mu \partial_\nu - m^2) \varphi = 0.$$

The solutions of this equation

$$u_{\vec{q}} = \frac{1}{\sqrt{2\omega_q(2\pi)^3}} \cdot e^{i\vec{q}\vec{x} - i\omega_q t}, \quad (2.17)$$

where

$$\omega_q \equiv \sqrt{\vec{q}^2 + m^2},$$

form the orthonormalizable basis. The modes (2.17), which are the eigenfunctions of operator $\partial/\partial t$ are the positive-frequency solutions with respect to t :

$$\frac{\partial}{\partial t} u_{\vec{q}}(t, \vec{x}) = -i\omega u_{\vec{q}}(t, \vec{x}).$$

To quantize φ one has to represent it as an operator satisfying the commutation relations

$$\begin{aligned} [\varphi(t, \vec{x}), \varphi(t, \vec{x}')] &= \left[\frac{\partial}{\partial t} \varphi(t, \vec{x}), \frac{\partial}{\partial t} \varphi(t, \vec{x}') \right] = 0; \\ \left[\varphi(t, \vec{x}), \frac{\partial}{\partial t} \varphi(t, \vec{x}') \right] &= i\delta(\vec{x} - \vec{x}'). \end{aligned}$$

Since the fields (2.17) form the orthonormalizable basis, the field φ can be represented as

$$\varphi(t, \vec{x}) = \int \frac{d^3 q}{\sqrt{(2\pi)^3 2\omega_q}} \left[u_{\vec{q}}(t, \vec{x}) c^-(\vec{q}) + u_{\vec{q}}^*(t, \vec{x}) c^+(\vec{q}) \right] \equiv \varphi^{(+)} + \varphi^{(-)}, \quad (2.18)$$

where operators $c(\vec{q})$ satisfy the commutation relations:

$$\begin{aligned} [c^-(\vec{q}), c^-(\vec{q}')] &= [c^+(\vec{q}), c^+(\vec{q}')] = 0, \\ [c^-(\vec{q}), c^+(\vec{q}')] &= (2\pi)^3 2\omega_q \delta^3(\vec{q} - \vec{q}'). \end{aligned} \quad (2.19)$$

The fields $\varphi^{(\pm)}$ are the positive- and negative-frequency parts of φ . Operators $c^+(\vec{q})$ and $c^-(\vec{q})$ have the meaning of creation- and annihilation-operators of scalar field quanta. In Heizenberg representation the quantum states form the Hilbert space. If one uses the Fock basis in this space, then operator $c^-(\vec{q})$ acts on the vacuum state $|0\rangle$ of this basis as

$$c^-(\vec{q})|0\rangle = 0.$$

The form of Hamilton operator in the representation of creation-annihilation operators is:

$$\mathcal{H} = \int \frac{d^3 q}{4(2\pi)^3} \left[c^+(\vec{q}) c^-(\vec{q}) + c^-(\vec{q}) c^+(\vec{q}) \right]. \quad (2.20)$$

Now, after we have pointed out some features of scalar field theory in Minkowski space, let us consider the field theory in FRW space.

The scalar fields in FRW space are described by Lagrange density

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \frac{\partial \varphi}{\partial x^\mu} \frac{\partial \varphi}{\partial x^\nu} - \frac{1}{2} \left(m^2 + \frac{R}{6} \right) \varphi^2,$$

where

$$R = -\frac{6\ddot{a}}{a^3}$$

is the scalar curvature corresponding to metric (2.3). Here \ddot{a} denotes the second derivative with respect to τ . The factor $1/6$ shows that the conformal coupling to gravity is chosen (see the previous subsection). The motion equation derived from this Lagrangian has the form:

$$\frac{1}{a^4} \frac{\partial}{\partial \tau} \left(a^2 \frac{\partial \varphi}{\partial \tau} \right) - \frac{1}{a^2} \Delta \varphi + \left(m^2 - \frac{\ddot{a}}{a^3} \right) \varphi = 0.$$

The solutions of this equation can be written as

$$\varphi(\tau, \vec{x}) = \frac{1}{(2\pi)^{3/2}a} \int u(\tau, \vec{p}) e^{i\vec{p}\vec{x}} d\vec{p}. \quad (2.21)$$

If one redefines the field as

$$\tilde{\varphi} = a\varphi,$$

then

$$\ddot{\tilde{\varphi}} - \Delta \tilde{\varphi} - m^2 a^2(\tau) \tilde{\varphi} = 0, \quad (2.22)$$

i.e. one yields the Klein-Gordon equation with the time-dependent mass. Substituting (2.21) into (2.22) one obtains for $u(\tau, \vec{p})$:

$$\ddot{u} + \omega^2(\tau)u = 0. \quad (2.23)$$

where

$$\omega^2(\tau) = p^2 + a^2(\tau)m^2.$$

Equation (2.23) reminds us the classical equation of harmonic oscillator with time-dependent frequency.

For the initial conditions

$$\begin{aligned} u^\pm(\tau_0) &= \frac{1}{\sqrt{\omega_0}}, \\ \dot{u}^\pm(\tau_0) &= \pm i\sqrt{\omega_0}, \\ \vec{p}^2 + m^2 a^2(\tau_0) &= \omega_0^2 \end{aligned} \quad (2.24)$$

equation (2.23) has a solution in the integral form [14] :

$$u^\pm(\tau, \vec{p}) = \frac{1}{\sqrt{\omega_0}} e^{\pm i\omega_0 \tau} + \frac{m^2}{\omega_0} \int_0^\tau d\tau' u^\pm(\tau', \vec{p}) V_m(\tau') \sin(\omega_0(\tau - \tau')), \quad (2.25)$$

where

$$V_m(\tau) = m^2 [a^2(0) - a^2(\tau)]$$

and for the simplicity $\tau_0 = 0$ is taken. Note, that in (2.24) and (2.25) $\tau_0 = 0$ means the time-moment, when the processes under investigation begin and u^\pm are the positive (negative) frequency solutions of (2.23).

Quantization of field φ proceeds via establishing the commutation relations on the spacelike hypersurfaces [14, 16]

$$\begin{aligned} [\varphi(y), \varphi(y')] &= 0, \\ [\partial_\mu \varphi(y) d\sigma^\mu(y), \partial_\nu \varphi(y') d\sigma^\nu(y')] &= 0, \\ \int_\Sigma [\varphi(y), \partial_\mu \varphi(y')] f(y') d\sigma^\mu(y') &= i f(y), \end{aligned} \quad (2.26)$$

where y is some point on hypersurface Σ . Then introducing operators of creation and annihilation $c^+(\vec{p}), c^-(\vec{p})$ with the usual commutation relations (2.19) one can write the solution (2.21) in the form [16]:

$$\begin{aligned} \varphi(\tau, \vec{x}) &= \frac{1}{(2\pi)^{3/2} a \sqrt{2}} \int d\vec{p} \frac{e^{i\vec{p}\vec{x}}}{\sqrt{1 - |\lambda(\tau, p)|^2}} \times \\ &\times \left[\left(u^+(\tau, \vec{p}) + \lambda(\tau, p) u^-(\tau, \vec{p}) \right) c^+(\vec{p}) + \left(u^-(\tau, -\vec{p}) + \lambda^*(\tau, p) u^+(\tau, -\vec{p}) \right) c^-(\vec{p}) \right] \end{aligned} \quad (2.27)$$

Here the functions $u^+(\tau, \vec{p}), u^-(\tau, \vec{p})$ satisfy (2.25), while $|\lambda(\tau, p)|$ is an arbitrary function with the only restriction $|\lambda(\tau, p)| < 1$. This function $\lambda(\tau, p) \neq 0$ is the new feature of quantization in the spaces with nonstationary metric. If one compares equations (2.27) and (2.18), one notices that the terms with λ mix the positive- and negative-frequency parts. This is connected with the fact that in such spaces energy is not conserved. In the Minkowski space-time the requirement of positiveness of Hamiltonian of the free field with nonzero mass leads to unique choice of Fock representation [16]. However in the space-time with time-dependent metric there is no energy as a generator of symmetry group, since there is no invariance in respect to time translations, and thus in such spaces the choice of representation of commutation relations (2.26) is ambiguous. It means that the choice of vacuum, as a state invariant under available symmetry group is also non-unique.

If one writes the Hamiltonian of such system by means of operators of creation and annihilation [16], then it will contain also the terms of type $c^+(\vec{p})c^+(-\vec{p}), c^-(\vec{p})c^-(-\vec{p})$ (compare with (2.20)). This means that Hamiltonian is not Hermittean operator. Even if $\lambda = 0$, what corresponds to usual Fock representation, Hamiltonian has no sense of the operator in Hilbert space. In this case the definition of particle as a quantum of energy has no sense, since nondiagonal Hamiltonian has no eigenvalues. (Only if $a^2(\tau) = \text{const}$ the "dangerous" terms vanish and Hamiltonian becomes Hermittean operator). The solution of this problem is that one must take $\lambda(\tau, p) \neq 0$ and choose it in such a way, that Hamiltonian shall have the diagonal form and "dangerous" terms will be cancelled. The choice $\lambda \neq 0$ corresponds to the choice of new orthonormalizable basis of solutions \tilde{u} and to Bogolubov transformations of creation and annihilation operators [16].

$$\begin{aligned} c^-(\vec{p}) &= \alpha_\lambda \tilde{c}^-(\vec{p}) + \beta_\lambda \tilde{c}^+(-\vec{p}); \\ c^+(\vec{p}) &= \alpha_\lambda^* \tilde{c}^+(\vec{p}) + \beta_\lambda^* \tilde{c}^-(-\vec{p}), \end{aligned}$$

where

$$\alpha_\lambda \equiv \frac{1}{\sqrt{1 - \lambda^2}}; \quad \beta_\lambda \equiv \frac{\lambda}{\sqrt{1 - \lambda^2}}.$$

Since $\lambda(\tau, p)$ is time-dependent, for each time-moment the particular Hamiltonian is determined as a Hermittean operator in Hylbert space where the corresponding commutation relations are established. To different time-moments correspond different $\lambda(\tau, p)$ and different vacua.

2.4 Short-time-interval approximation

As we have seen, in the curved space-time, in general, there is no natural definition of what are the particles since particle creation caused by cosmological expansion takes place. However since the field theory works in Minkowski space quite successfully, there must be some approximation in the curved space-time, where the definition of particles has a sense. There must be the method how to choose those exact modes of motion equation solutions which in some sence are the mostly close to the corresponding solutions in the limit of Minkowski space. Physically this means the construction of such minimal disturbance of field in the expanding Universe, when the particles are defined so that the particle creation due caused by changing of geometry is minimal.

For example, one can investigate some process during sufficiently short time interval when the geometry of space-time does not change significantly. Then we can assume that λ is constant and construct the corresponding Hamiltonian.

In the short-time-interval approximation

$$a^2(\tau) \simeq 1 + 2\dot{a}(0)\tau = 1 + 2H\tau,$$

where it is taken $a(0) = 1$ and H is Hubble constant. The correct choice of λ from (2.27) which diagonalizes the Hamiltonian gives [16]

$$\lambda(\tau, p) = \frac{m^2 H}{2\omega^3} e^{i\omega\tau} \sin \omega\tau.$$

It is clear that $\lambda \equiv 0$ when $H = 0$.

The Hamiltonian depends on Hubble parameter and in the approximation with zero Hubble parameter one yields the usual field theory in Minkowski space.

Field quantization in FRW space leads to some physical consequences.

In particular:

1. Since $\lambda(\tau, p)$ is complex, T -invariance is violated in corresponding field theory.
2. If in some moment the particles were described by functions $\varphi(\tau, \vec{x})$ so that $\varphi^-(\tau, \vec{x})|0\rangle = 0$, where $|0\rangle$ is vacuum state, then in the next moment the "anomalous vacuum expectation values" in respect to new vacuum $|0'\rangle$ can appear [16]:

$$\langle 0' | \varphi^-(\tau', \vec{x}) \varphi^-(\tau', \vec{x}) | 0' \rangle \neq 0, \quad \langle 0' | \varphi^+(\tau', \vec{x}) \varphi^+(\tau', \vec{x}) | 0' \rangle \neq 0.$$

This violates gauge invariance and thus the charge conservation. This effect violates the conservation of any charge (among them the Baryon charge) [16]. The value of this violation can be estimated :

$$\langle 0' | \varphi^+ \varphi^+ | 0' \rangle = (\langle 0' | \varphi^- \varphi^- | 0' \rangle)^* \leq \frac{mH}{8(2\pi)^3} e^{im\tau} \sin m\tau.$$

However one has to note that CP -violation caused by the Universe expansion occurs only if Bogolubov transformations affect particles but not antiparticles. This means that only matter but not antimatter "feels" the expansion of the universe and antiparticles do not interact with external gravitational field. Such model was discussed in [30].

3. In the FRW Universe the nonzero classical density of particle number can appear:

$$n_0 = i \langle 0' | \dot{\varphi}^+ \dot{\varphi}^- - \dot{\varphi}^+ \varphi^- | 0' \rangle = \frac{mH^2}{16(2\pi)^3} \sin^2 m\tau \simeq 10^{-46} \text{cm}^3.$$

This phenomena inspite of its small value could play essential role at the early stages of Universe expansion.

Now let us show that in the short-time-interval approximation the definition of particles can have the usual meaning.

The formal solution of equation (2.23), which reminds us the classical equation of harmonic oscillator with time-dependent frequency, can be written also in the form

$$u(\tau) = \frac{1}{\sqrt{2W}} \cdot e^{-\int_{\tau} W(\xi) d\xi},$$

where the function W satisfies the nonlinear equation

$$W^2(\tau) = \omega^2(\tau) - \frac{1}{2} \left(\frac{\ddot{W}}{W} - \frac{3}{2} \frac{\dot{W}^2}{W^2} \right).$$

One can expand $W(\tau)$ into series in terms of adiabatic parameter of the oscillator

$$\epsilon = \frac{\dot{\omega}}{\omega^2} = \frac{m^2}{\omega^3} \cdot a\dot{a}.$$

The zero order adiabatic approximation will be

$$W^0(\tau) \simeq \omega(\tau),$$

which leads to standard modes of Minkowski space.

The maximal value for ϵ when the momentum $p = 0$ is

$$\epsilon_{max} = \frac{\dot{a}}{a^2 m} = \frac{H}{m}.$$

The effect of particle creation is significant when $\epsilon \sim 1$.

If we consider the GUT scale, when $a \sim 1/M_X$ (M_X is the X -boson mass), and take a short interval of time

$$t \sim \frac{1}{10M_X}, \quad \tau \sim \frac{1}{10}, \quad (2.28)$$

we can decompose $\omega(\tau)$ into series in terms of τ :

$$\omega = \sqrt{p^2 + a^2(\tau)m^2} \simeq \omega_0(1 + \epsilon\omega_0\tau),$$

where $\omega_0 = \sqrt{p^2 + a_0^2 m^2}$ and

$$\epsilon = \frac{a_0 \dot{a}_0 m^2}{\omega_0^3}.$$

One can show that adiabatic parameter is of the order of

$$\epsilon \sim \frac{M_X}{M_{Pl}} \sim 10^{-4},$$

so we can neglect particle creation and such decomposition is valid. Certainly, for de Sitter expansion law (see (2.8))

$$\begin{aligned} a &= \frac{a_0}{1 - K\sqrt{\rho}a_0\tau}; \\ \dot{a}_0 &= a_0 K\sqrt{\rho}, \end{aligned} \tag{2.29}$$

where the constant K is described by formula (2.7), we have $\sqrt{\rho} \sim M_X^2$, $\sqrt{G} \sim 1/M_{Pl}$ and

$$\epsilon \sim \sqrt{G\rho}/M_X \sim M_X/M_{Pl}.$$

For the power-function low expansion (2.9)

$$\begin{aligned} a &= a_0 + K\tau; \\ \dot{a}_0 &= K, \end{aligned} \tag{2.30}$$

we have

$$\epsilon \sim \frac{\sqrt{G}}{a_0^2 M_X} \sim \frac{M_X}{M_{Pl}}$$

again.

In the short-time-interval approximation solution of (2.22) has the form

$$\varphi = \left(\frac{1}{\sqrt{2}\omega_0} - \epsilon \frac{\sqrt{2}\omega_0}{8} \tau \right) \cdot e^{i\omega_0\tau(1 + \frac{\omega_0}{2}\epsilon\tau)}.$$

For such a small values of ϵ and τ in this formula one can neglect the terms containing $\epsilon\tau$. Thus for the sufficiently short time intervals we have the usual definition of positive- and negative-frequency states and we can consider the usual definition of particles by means of creation-annihilation operators.

Chapter 3

Effects of CPT -violation in FRW Universe

At present there is good evidence that each of the three discrete symmetries C , P and T by itself is only approximately valid in the particle theory. The same applies to any of the bilinear products of these symmetries. However the triple product CPT is considered to be exact symmetry. Hence for elementary particle physics usually regains symmetry when we interchange particles with antiparticles, right with left and past with future. Thus, theories of CPT violation must necessarily step outside the standard theory.

Developments in the quantum theory of gravity have opened possible way to theories of CPT violation. Building from his results on the spectrum of radiation from black holes, Hawking has proposed that the generalization of quantum mechanics which encompasses gravity allows the evolution of pure states into mixed states [31]. Then it was shown that any such dynamics leads to conflict with CPT conservation (e.g. [32]). These ideas raised the interesting possibility that one could find observable CPT violation due to a mechanism that lies not only beyond a local quantum description but also beyond quantum mechanics altogether. The notion that gravitation effects beyond quantum mechanics can effect elementary particle physics is controversial. For example, in theories which allow the evolution of pure states into mixed states there is a serious conflict between energy-momentum conservation and locality [33].

One of the basic assumptions in proving of CPT -theorem is the invariance of theory under continuous Lorentz transformations. Thus in arbitrary gravitational field CPT needs not to be realized as a compulsory symmetry. For example, the Universe expansion violates both — Lorentz invariance and time-reversal and will cause the violation of CPT . In this case there is no reason to assume that particles and antiparticles inevitably must have the same masses and lifetimes.

In this section we consider the effects of CPT -violation in FRW spaces.

3.1 CPT -theorem in Minkowski space

In order to understand the effects of CPT -violation in the expanding Universe let us briefly discuss some consequences of CPT -conservation in Minkowski space

[7, 34, 35].

In Minkowski space for any spin- $\frac{1}{2}$ field the discrete symmetry transformation can be defined as

$$\begin{aligned} T\Psi(\vec{x}, t)T^{-1} &= \eta_t \gamma_1 \gamma_3 \Psi(\vec{x}, -t); \\ P\Psi(\vec{x}, t)P^{-1} &= \eta_p \gamma_0 \Psi(-\vec{x}, t); \\ C\Psi_\alpha(\vec{x}, t)C^{-1} &= \eta_c (\gamma_2)_{\alpha\beta} \Psi_\beta^\dagger, \end{aligned} \quad (3.1)$$

where η_t , η_p and η_c are constant phase factors $|\eta| = 1$ (the last relation is written in components). For the *CPT* transformation, operator of which is defined as

$$\Theta = CPT, \quad (3.2)$$

we obtain

$$\Theta\Psi_\alpha(\vec{x}, t)\Theta^{-1} = \eta(i\gamma_5)_{\alpha\beta} \Psi_\beta^\dagger(-\vec{x}, -t), \quad (3.3)$$

where η is the product of arbitrary phase factors η_t , η_c and η_p .

For the boson fields $\varphi(x)$ we have

$$\Theta\varphi(x)\Theta^{-1} = \eta\varphi^\dagger(-x). \quad (3.4)$$

To demonstrate how the *CPT* theorem is proved, let us consider a local field theory with different spin fields and demand that the theory must be invariant under continuous Lorentz group. All these fields satisfy the usual spin-statistics relation: integer-spin fields obey Bose statistics, while half-integer-spin fields obey Fermi statistics.

If we assume that the theory can be described by a local n -th rank Lorentz-tensor $F_{k_1 \dots k_n}$ which is a normal product of fields, then considering all possible field-combinations and using (3.3), (3.4) one can show that [7]

$$\Theta F(x)\Theta^{-1} = (-1)^n \cdot F^\dagger(-x).$$

The Lagrangian density L is one of such products and it is a tensor of 0-th rank. Therefore setting $n = 0$ we obtain

$$\Theta L(x)\Theta^{-1} = L^\dagger(-x). \quad (3.5)$$

In a quantum field theory the Lagrangian is a Hermitean operator $L = L^\dagger$. The action is the 4-dimensional volume integral $S = \int L d^4x$. Using (3.5) we see that S is invariant under *CPT* transformation. Thus the theory and the corresponding Hamiltonian is invariant under *CPT* :

$$\Theta \mathcal{H} \Theta^{-1} = \mathcal{H}. \quad (3.6)$$

In proving the *CPT*-theorem the following assumptions are used: (a). Invariance under the continuous Lorentz transformations; (b). spin-statistical relations and (c) the locality of field theory.

Now let us consider a massive particle ψ at rest. Let the state $|\psi\rangle_l$ be the one with its z -component angular momentum equal to l . Apart from a multiplicative phase factor the state changes under the *C*-operation in the following way:

$$C|\psi\rangle_l \rightarrow |\bar{\psi}\rangle_l.$$

Under P -operation it remains itself :

$$P|\bar{\psi} >_l \rightarrow |\bar{\psi} >_l,$$

but under T , since l changes its sign,

$$T|\bar{\psi} >_l \rightarrow |\bar{\psi} >_{-l}.$$

Therefore the CPT transformation (3.2) makes the following changes:

$$\Theta|\psi >_l = e^{i\alpha}|\bar{\psi} >_{-l}, \quad (3.7)$$

where α is constant phase factor.

The mass of the particle ψ is given by the expectation value

$$m = \langle \psi | \mathcal{H} | \psi \rangle_l, \quad (3.8)$$

where \mathcal{H} is the total Hamiltonian, clearly real and independent on l . Since (3.8) equals to its complex conjugate, one can write

$$m = \langle \psi | \mathcal{H} | \psi \rangle_l^* = \langle \psi | \Theta^{-1} \Theta \mathcal{H} \Theta^{-1} \Theta | \psi \rangle_l.$$

Due to (3.6) and (3.7) the above expression can also be written as

$$m = \langle \bar{\psi} | \mathcal{H} | \bar{\psi} \rangle_{-l} = \bar{m}. \quad (3.9)$$

Thus the CPT -invariance of Hamiltonian leads to the mass equality between particles and antiparticles.

The other application of CPT -theorem is the lifetime equality between particles and antiparticles.

Let us consider the decay of particle ψ and its antiparticle $\bar{\psi}$ through some interaction Hamiltonian \mathcal{H}_{int} . To the lowest order of \mathcal{H}_{int} the decay-widths (the inverse quantities to the lifetimes) of ψ and $\bar{\psi}$ are given by the perturbation formulae

$$\Gamma_\psi = 2\pi \sum_f \delta(E_f - E_i) |\langle f | G(\infty, 0) \mathcal{H}_{int} | i \rangle|^2;$$

$$\Gamma_{\bar{\psi}} = 2\pi \sum_{\bar{f}} \delta(E_{\bar{f}} - E_{\bar{i}}) |\langle \bar{f} | G(\infty, 0) \mathcal{H}_{int} | \bar{i} \rangle|^2,$$

where $|i\rangle$ and $|f\rangle$ are the initial and the final states and $G(t, t_0)$ is Green's function of motion equation in the representation of interaction. If CPT -invariance holds, then

$$\Theta G(t, t_0) \Theta^{-1} = G(-t, -t_0).$$

Using this formula we can convert the above expression for Γ_ψ into

$$\begin{aligned} \Gamma_\psi &= 2\pi \sum_f \delta(E_f - E_i) \left| \langle f | \Theta^{-1} \Theta G(\infty, 0) \Theta^{-1} \Theta \mathcal{H}_{int} \Theta^{-1} \Theta | i \rangle \right|^2 = \\ &= 2\pi \sum_f \delta(E_f - E_i) \left| \langle \bar{f} | G(-\infty, 0) \mathcal{H}_{int} | \bar{i} \rangle \right|^2. \end{aligned}$$

Using definition of S -matrix

$$G(-\infty, 0) = S^\dagger G(\infty, 0)$$

and the relations between the initial and the final energies

$$E_f = E_{\bar{f}},$$

$$E_i = E_{\bar{i}},$$

we can write the equation of decay-width in the form

$$\begin{aligned} \Gamma_\psi &= 2\pi \sum_{\bar{f}} \delta(E_{\bar{f}} - E_{\bar{i}}) \left| \langle \bar{f} | S^\dagger G(\infty, 0) \mathcal{H}_{int} | \bar{i} \rangle \right|^2 = \\ &= 2\pi \sum_{\bar{f}} \delta(E_{\bar{f}} - E_{\bar{i}}) \left| \sum_{\bar{f}'} \langle \bar{f} | S^\dagger | \bar{f}' \rangle \langle \bar{f}' | G(\infty, 0) \mathcal{H}_{int} | \bar{i} \rangle \right|^2. \end{aligned}$$

Since $S^\dagger S = 1$ and the S -matrix has the only nonvanishing matrix elements between the equal-energy states [36], we have

$$\sum_{\bar{f}} \delta(E_{\bar{f}} - E_{\bar{i}}) \langle \bar{f} | S^\dagger | \bar{f}' \rangle \langle \bar{f} | S^\dagger | \bar{f}'' \rangle = \delta(E_{\bar{f}'} - E_{\bar{i}}) \delta_{\bar{f}' \bar{f}''}.$$

Using this formula it is easy to see that

$$\Gamma_\psi = 2\pi \sum_{\bar{f}'} \delta(E_{\bar{f}'} - E_{\bar{i}}) \left| \langle \bar{f}' | G(\infty, 0) \mathcal{H}_{int} | \bar{i} \rangle \right|^2 = \Gamma_{\bar{\psi}}.$$

This establishes the lifetime equality between ψ and $\bar{\psi}$ to the lowest order of the decay Hamiltonian \mathcal{H}_{int} .

There are several tests which check the CPT invariance. The most simple ones are the equality of masses and lifetimes of a particles and antiparticles [37]. The best experimental support of CPT -invariance at the present stage of the Universe expansion is given from such identity for K -mesons [6]. Existing experimental evidence for CPT invariance is poor, e.g. the limit for the strength of the CPT -violating interaction is only 1/10 of the CP -violating interaction [38].

3.2 The direct- and inverse-decay rate difference

Now let us discuss some effects of CPT -violation in strong gravitational fields.

Parker [15] concerned with particle creation by the gravitational field and its possible applications to cosmology while in [17] some effects of interactions were considered. Attention was paid also to the possibility of CPT -violation in the presence of a time-dependent gravitational field which breaks Lorentz-invariance. Brout and Englert [18] have discussed the possibility of CPT -violation induced by the expansion of the Universe. However in the particular model which they investigated such a violation does not in fact occur. Barshay [19] had also discussed the effects of a phenomenological CPT -violating interaction in the early Universe.

In papers of Ford [20], Grib and Kryukov [24] and Lotze [23] the effect of expansion of the Universe upon the rate of decay of a massive particles was investigated. There was argued that the average decay rate of particle in general is not CPT -invariant. Authors calculated the difference between the direct and inverse decay rates in the background of FRW universe. This difference and the consequent CPT violation arises because the expansion of the Universe causes a mixing of positive and negative frequency modes (see subsection 2.3); it is this mixing which is responsible for both particle creation [15] and CPT violation. Brout and Englert [18] do not find CPT violation in their models because they worked in the approximation where the mixing of positive and negative frequencies is ignored.

Ford had derived that the difference between the rate of decay of massive scalar particle Φ with momentum \vec{p} into n massless particles φ and the rate of inverse decay depends on a quantity

$$\Delta_{CPT} = \frac{1}{\omega \cdot \Delta\tau} \left[\int_{-\infty}^{+\infty} d\tau a^{3-n}(\tau) \sin(2\omega\tau) \cdot \int_{-\infty}^{+\infty} dy \cos(2\omega y) V(y) - \int_{-\infty}^{+\infty} d\tau a^{3-n}(\tau) \cos(2\omega\tau) \cdot \int_{-\infty}^{+\infty} dy \sin(2\omega y) V(y) \right] \quad (3.10)$$

where

$$\omega = \sqrt{(\vec{p})^2 + m^2 a^2(\pm\infty)},$$

$$V(\tau) = m^2 [a^2(\pm\infty) - a^2(\tau)] \quad (3.11)$$

(compare with (2.25)) and it is assumed that

$$a(+\infty) = a(-\infty). \quad (3.12)$$

The quantity (3.10) is a measure of CPT -violation. Note that if $a(\tau)$ is an even function or constant, then Δ_{CPT} vanishes. Similarly, if one lets $a(\tau) \rightarrow a(-\tau)$, then (3.10) changes its sign. Thus CPT -invariance is preserved in a generalized sense: the average decay rate in a Universe is equal to the average inverse decay in the Universe with the time reversed scale factor. Note also, that (3.10) is equal to zero if $n = 3$. Thus CPT violation is possible in this model only if the interaction is not conformally invariant.

Consequently Ford required three conditions to be fulfilled simultaneously in order that the interaction of scalar particles considered by him violated CPT -invariance:

1. The conformal scale factor must be an even function of the conformal time parameter.
2. The conditions for particle creation in non-interacting theory have to be fulfilled for at least one of the quantum fields participating in the interaction.
3. Additionally, the interaction itself has to break conformal invariance.

More generalized case of the decay of massive scalar particle Φ into n massive scalar particles φ was considered by Grib and Kryukov [24]. They obtained that the difference between the rates of decay and inverse decay are different in this case even if $n = 3$ (however again $a(\tau)$ must be not time-even), since in this case the interaction is not conformally invariant.

Lotze [23] has calculated the rates of $\pi^0 \rightarrow \gamma\gamma$ and $\gamma \rightarrow e^+e^-$ decays and inverse decays in the background of time-asymmetrically expanding Universe. He derived, that *CPT* is violated in this decays even though the interaction of photons and fermions does not change the conformal-invariance properties of the theory. Thus the third condition is not necessary in this case.

In all the cases the effect of *CPT* violation is caused by the Universe expansion (collapse), which leads to the mixing of positive and negative frequencies at the exponents in the solutions of the field equation; i.e. the effect of *CPT* violation is caused by the coefficients of the exponents due to frequency mixing. This coefficients (at the exponents with mixed frequencies) are responsible also for particle creation by the gravitational field. This means that Characteristic time, when the effect is significant, is the Compton time (and not the Planck time) determined by particle masses. One can derive that Δ_{CPT} vanishes when $\Delta\tau \cdot \omega \rightarrow \infty$. This reflects the fact that the wavelengths much shorter than the local radius of curvature of space-time are not effected by the background gravitational field. I.e. *CPT*-violation is the most noticeable when the particle's wavelength is of the order of the radius of curvature.

It is clear that in the absence of gravitational field (i.e. in the case of Minkowski space-time) the effect of *CPT*-violation is absent, while in the presence of non-stationary gravitational field, which is not invariant in respect to time-reflection (this is the case in FRW cosmology) the effect of *CPT*-violation is nonzero. Thus the measure of *CPT*-violation can be associated with the parameter describing intensity of gravitational field. Such the parameter is Hubble parameter $H = \dot{a}(\tau)/a^2(\tau)$, which at present is equal to $H \sim 6 \cdot 10^{-27} \text{cm}^{-1}$.

Let us show, that the measure of some effect caused by *CPT* violation due to variation of $a(\tau)$ during the time interval $\Delta\tau$ beginning from some initial moment τ_0 is proportional to $m^2 H$ (where H is the value of Hubble parameter for the time interval under consideration). In particular, τ_0 can be also a singular point.

For the phenomena considered in papers [20, 23, 24] the measure of *CPT* violation is determined by formula (3.10) which includes the quantity (3.11). It is clear that $V(\tau)/\Delta\tau \sim (a^2(\pm\infty) - a^2(\tau))/\Delta\tau \sim \dot{a}(\tau)|_{\tau=c}$, where c is some moment in the considered time interval $\Delta\tau$. Since for the models with the condition (3.12) the quantity $1/a^2(\tau)$ always can be evaluated by some finite number, the whole expression (3.10) is proportional to $m^2 H$.

One can show that the measure of *CPT*-violation is proportional to $m^2 H$ also in other models including the models with singularities (see [24]). This means that the effect is significant for Compton times.

3.3 Particle-antiparticle mass-difference

The other consequence of *CPT*-violation caused by the Universe expansion can be the mass-difference between particles and antiparticles [25].

In subsection 2.2 we have seen that when the curvature of FRW metric is taken into account via conformal transformations of field operators, the scalar, spinor and vector fields behave in different manner. This difference can be exhibited when one investigates the particle interactions in FRW space. In particular, taking into account the formulae of conformal transformations (see subsection 2.2) it is easy to find that

the mass acquired by initially massless vector particles (e.g. leptoquarks) through the Higgs mechanism becomes time-dependent [26]:

$$M_X^2 = a^2(\tau)g < \varphi > . \quad (3.13)$$

Here g is the Yukawa coupling constant and $< \varphi >$ is the vacuum expectation value of the Higgs field.

Here arises the problem, how to define, what is particle and what is antiparticle, since equation (3.13) corresponds to the classical equation for the harmonic oscillator with time-dependent frequency

$$\omega^2(\tau) = p^2 + M_X^2(\tau)$$

and in this case the time reflection (opposite to flat-wave case) does no longer transfer the eigenvalue with positive frequency into eigenvalue with negative frequency, i.e. particle into antiparticle.

This problem can be avoided if the particles acquire their masses during sufficiently short period

$$\tau \ll 1.$$

Then it is possible to use the short-time-interval approximation (see subsection 2.4). Let us decompose $a(\tau)$ into a series in τ and leave only the first order in the wave equation. For the time part of wave function we obtain the flat-wave expression $\exp(i\omega_0\tau)$, where

$$\omega_0 = (p^2 + m_X^2)^{1/2},$$

and

$$m_X^2 = a_0^2 < \varphi >^2 .$$

In this approximation the frequency ω_0 does not depend on time and the usual definition of antiparticles (as a particles with negative frequency or "moving backwards" in time) and the form of CPT -transformation operators (3.2) remains valid. Corrections connected with the time dependence of metric remain only in the expression of the Hamiltonian of interactions (see subsection 2.2) and in the mass of leptoquarks. In the first order of τ

$$M_X(\tau) \sim m_X + \Delta m_X,$$

where

$$\Delta m_X = (\dot{a}|_{\tau=0})m_X\tau/a_0 \quad (3.14)$$

(the point means the derivative with respect to τ).

For the mass of antileptoquarks (obtained by the time reflection $\tau \Rightarrow -\tau$) we have:

$$M_{\bar{X}}(\tau) \sim m_X - \Delta m_X.$$

In the expanding Universe Δm_X is positive ($\dot{a} > 0$) and the mass of leptoquarks is larger then the mass of antileptoquarks. Below we shall see that this leads to the predominance of matter over antimatter. In the collapsing Universe Δm_X is negative since $\dot{a} < 0$ and we should observe the opposite picture.

Herefrom we can conclude that the sign of BAU and the direction of cosmological arrow of time are correlated.

Chapter 4

Cosmological arrow of time and BAU

Different asymmetry features of Nature are, seemingly, connected with each other. We shall see that Baryon Asymmetry of the Universe is correlated with the other asymmetry — cosmological time arrow. Cosmological expansion causes violation of T - and CPT -symmetry, while the latter provides the mass-difference between particles and antiparticles. In this case even in thermodynamical equilibrium there can be the predominance of matter over antimatter. This means that the usual scenario of BAU generation changes radically.

In this section we shall calculate the BAU generated on GUT scale in thermal equilibrium when CPT is violated due to the Universe expansion.

4.1 The problem of baryon asymmetry

One of the basic observational facts in cosmology is the apparent predominance of matter over antimatter.

The Baryon Asymmetry of the Universe (BAU) can be described by the quantity

$$\Delta(t) = \frac{n_b(t) - n_{\bar{b}}(t)}{n_b(t) + n_{\bar{b}}(t)},$$

where $n_b(t)$ and $n_{\bar{b}}(t)$ are the densities of baryons and antibaryons at the moment t . At present ($t = 3 \cdot 10^{17} \text{sec}$) this quantity $\Delta(3 \cdot 10^{17} \text{sec}) \approx 1$, since $n_{\bar{b}} \approx 0$. At the temperatures $T > m_{nucl}$, i.e. $t < 10^{-6} \text{sec}$, the amount of nucleon-antinucleon (quark-antiquark) pairs in plasma is equal to amount of photons (the difference is a factor ~ 1). So

$$\Delta(t \leq 10^{-6} \text{sec}) \sim \frac{n_b - n_{\bar{b}}}{n_\gamma} \equiv \delta,$$

where n_γ is the photon density. As the Universe expands isentropically, the quantity δ changes with time very weakly (it changes by factor ~ 5 due to photon gas heating through the heavy particle annihilation), since at $T \sim 1 \text{GeV} \sim 10^{13} \text{K}$ the rate of

possible B-violating processes is negligible. Thus

$$\delta = \frac{n_b - n_{\bar{b}}}{n_\gamma} \Big|_{t \leq 10^{-6} \text{ sec}} \approx \frac{n_b}{n_\gamma} \Big|_{t = 3 \cdot 10^{17} \text{ sec}}.$$

Essentially the photons in the Universe are in 3 K background; the number density of photons is $n_\gamma = 2\zeta(3)/(\pi^2 T_\gamma^3)$, where $\zeta(3) \simeq 1.202$ and $T_\gamma \simeq 2.9K$.

The present baryon density n_b is conveniently expressed in terms of the parameters h (Hubble parameter $H \equiv 100h \text{ km} \cdot \text{sec}^{-1} \cdot \text{Mpc}^{-1}$) and $\Omega = \rho_b/\rho_c$ ($\rho_c = 3H^2/(8\pi G)$ is the critical density of the Universe [11] and $\rho_b = m_b \cdot n_b$ is the baryon mass density):

$$n_b = \Omega h^2 \cdot 1.13 \cdot 10^{-5} \text{ cm}^{-3}.$$

This is also the net baryon number density n_B . Our knowledge of h and Ω is poor; we take as extreme limits $1/2 < h < 1$ and $0.005 < \Omega < 2$. (The upper limit $\Omega = 2$ corresponds to a deceleration parameter $q_0 = 1$, while galactic masses determined from rotation curves indicate $\Omega > 0.005$). With these ranges we find that the baryon-per-photon ratio is (see e.g. [39]):

$$\delta = \frac{n_B}{n_\gamma} \sim 10^{-11} \div 10^{-8}. \quad (4.1)$$

Very often, instead of (4.1), the baryon number density to entropy density ratio n_B/s is considered. Approximately $n_B/s \approx 1.7n_B/n_\gamma$ (see [39]). The entropy s per comoving volume is a more useful quantity because s is a constant for an adiabatically expanding Universe, while, as we mentioned above, n_γ is modified by processes such as e^+e^- annihilation.

The quantity δ is one of the fundamental quantities of the Universe.

Different kinds of models, trying to explain the origin of BAU and value of δ were constructed (for the review see [40, 41]).

If the baryon number were conserved, the observed baryon excess would probably have to be postulated as an initial condition on the big bang. The requirement of such a small number as an arbitrary initial condition is possible but very unesthetic. The other possibility, that the total baryon number of the Universe is zero but baryons and antibaryons exist in widely separated regions, is generally disfavored (for the references see e.g. [40, 41]).

With the advent of GUTs it became clear that the baryon number is not conserved and BAU can be generated dynamically in the first instant after big bang, while the initial baryon number of the Universe is zero or an arbitrary non-zero value. Various mechanisms of baryogenesis were constructed such as those involving quantum gravity, particle creation in the gravitational field of an expanding Universe, the creation and evaporation of black holes and monopoles (see also [42]), the sufficient entropy generation in initially cold charge-asymmetric Universe, the effects of anisotropy or inhomogeneities, the baryon number generation in theories with integer charged quarks and so on. The reviews of these mechanisms and references are given in [40, 41, 43, 44, 45].

The most popular type of models (which we will discuss here) was proposed first by Sakharov [46] and Kuzmin [47], who showed that baryon number generation requires three conditions:

(a). Some interactions of elementary particles must violate baryon number, since the net baryon number of the Universe must change over time.

(b). CP must be violated. This will provide an arrow for the direction of B -violation and the rates of the B -violating processes will be not equal: $\Gamma(i \rightarrow f) \neq \Gamma(\bar{i} \rightarrow \bar{f})$, where i and f denote the initial and final particles.

(c). A departure from thermal equilibrium must take place. Certainly, if a symmetrical Universe is in thermal equilibrium, particle number densities behave as $\exp(-m/T)$. CPT invariance guarantees that a particle and its antiparticle have the same mass, and unitarity requires that the total production rates of a particle and its antiparticle be equal, so their densities remain equal during expansion and no asymmetry arises, regardless of B -, C - and CP -violating interactions (see e.g. [40, 48]).

GUTs provide (a) due to interactions of quarks and leptons with heavy bosons (For the minimal $SU(5)$ GUT this interactions are $X \leftrightarrow UU$, $X \leftrightarrow \bar{D}\bar{L}$, $Y \leftrightarrow UD$, $Y \leftrightarrow \bar{U}\bar{L}$, $Y \leftrightarrow \bar{D}\bar{\nu}$, $H_t \leftrightarrow \bar{U}\bar{D}$, $H_t \leftrightarrow UL$, $H_t \leftrightarrow D\nu$, where X, Y are leptoquarks, H_t — Higgs color triplet, U — up-like quarks, D — down-like quarks, L — charged leptons and we used the notation of [41], opposite to [39, 49]) and they can naturally contain (b) through loop processes (see [49, 50]), while (c) arises in the early Universe, when particle reaction rates Γ lag behind the rate of cosmological expansion H .

The scenario GUT-scale creation of BAU (see e.g. [39, 40, 41, 48, 49, 51, 52, 53, 54]) is as follows.

Soon after big bang the baryon number violating interactions came into equilibrium so that any initial baryon number was washed out. After that, when the temperature fell down below the value M_χ (where $\chi \equiv X, Y \text{ or } H_t$), the decay rates of heavy bosons became less than the rate of Universe expansion and B -violating reactions dropped out of equilibrium. The subsequent decays of heavy bosons (inverse decays were blocked since typical particles had energies $\sim T < M_\chi$) have generated the BAU. This could have occurred despite the equal densities and the equal total decay rates (which follow from CPT) of the χ and $\bar{\chi}$, because if C and CP were violated, one could still have different partial decay rates:

$$\Gamma(\chi \rightarrow f) \neq \Gamma(\bar{\chi} \rightarrow \bar{f}).$$

After these decays baryon number was effectively conserved until today, as rates of B -violating processes are very small for $T \ll M_\chi$, so this baryon excess remained constant. In the and, as the time passed, the charge-symmetrical part of plasma annihilated into photons, neutrinos and anti-neutrinos, while the "surplus" particles survived and originated the observed world.

The resulting baryon-to-photon ratio is [39, 41] :

$$\delta = \left(\frac{n_B}{n_\gamma} \right)_{\text{initial}} \cdot e^{-\xi A} + \frac{b}{1 + (cK)^d} \cdot \Delta B_\chi, \quad (4.2)$$

where

$$\Delta B_\chi = \sum_f B_f \cdot \frac{\Gamma(\chi \rightarrow f) - \Gamma(\bar{\chi} \rightarrow \bar{f})}{\Gamma_{\text{total}}(\chi)},$$

is the total baryon number created through χ decays; parameter

$$A = \left(16\pi^3 \frac{g_*}{45}\right)^{-1/2} \cdot \frac{\alpha M_{Pl}}{M_\chi} \sim \left(\frac{\Gamma}{H}\right)$$

describes the value of nonequilibrium; B_f is the baryon number of the final state f ; parameters $\xi = 5.5(0.25 \div 1.6)$, $b = 0.03(0.01)$, $c = 16(3)$, $d = 1.3(1.2)$ for $\chi \equiv XY(H_t)$; α_χ is the coupling constant; $g_* = 160$ is the "effective" complete (sum over bosons and 7/8 times the sum over fermions) number of degrees of freedom in relativistic particles.

In (4.2) the first term describes the washout of initial asymmetry (if such had existed), while the second term — creation of new asymmetry.

In the different models the X and Y or H_t bosons were used as a source of BAU. Some models considered washout of initial asymmetry by X, Y and consequent generation of BAU by H_t bosons (see e.g. [39, 41]).

However in the minimal $SU(5)$ model the natural choice of parameters $M_{XY} \sim (10^{15} \div 10^{14})GeV$, $M_{H_t} \sim 10^{13}GeV$ requires $\Delta B_{XY} \sim 10^{-5.7}$ and $\Delta B_{H_t} \sim 10^{-8.5}$, while in this model $\Delta B_\chi < 10^{-10}$ [49, 51]. Thus the minimal $SU(5)$ model fails to explain the observable BAU and one needs to consider different extended models, for example those with extra Higgses (see also [55]).

Another difficulty of GUT-scale baryogenesis models is that in the standard model of electroweak interactions baryon number is known theoretically to be anomalous and not exactly conserved [56]. At low temperature this anomalous baryon violation only proceeds via an exponentially suppressed tunneling process, at a rate proportional to $\exp(-4\pi/\alpha_w) \sim 0$, where α_w is the weak coupling constant. At temperatures above $\sim 100GeV$, however, electroweak baryon violation may proceed rapidly enough to equilibrate to zero any baryons produced by Grand Unification [57].

In the last decade much work has been done to solve this problem.

One of the most popular fields of investigations opened in 1985 by the work of Kuzmin, Rubakov and Shaposhnikov [58], who showed that at the scale of electroweak phase transitions there are all three necessary ingredients for baryogenesis (B -violation via anomalous processes, CP -violation in Kobayashi-Maskawa matrix, departure from thermal equilibrium if the electroweak phase transition is of the first order) and BAU can be created at this scale. After that many mechanisms of electroweak-scale baryogenesis were proposed (for the review see [59, 60, 62, 63]).

Most of these mechanisms consider different extensions of standard model (such as singlet Majoron model with complex phases in the neutrino mass matrix; the nonsupersymmetric two Higgs doublet model with a CP -violating relative phase between the doublets; and minimal supersymmetric standard model with a CP -violating phase in the gaugino masses), since CP -violation from Kobayashi-Maskawa matrix is by many orders small to give the observed value for δ and one has to search for additional source of CP -violation. Besides, there arise problems with the mass of higgs particles (see [60, 62]) - the schemes of baryogenesis in the Minimal Standard Model require the Higgs mass to be less than $50GeV$ that is forbidden experimentally. Nevertheless, there are some attempts to describe the BAU via electroweak-scale processes in the frames of Minimal Standard Model [60, 64] (some problems of the model proposed in [60] are discussed in [61]). In [64] a new source of CP -violation comes

from P -violation by the domain walls, appearing in the first-order electroweak phase transitions, while the problem of higgs mass is solved due to the mass-difference in the leptonic sector (see [65]).

In the other set of works it was noted, that anomalous electroweak processes conserve $B - L$, and so a net $B - L$ generated by GUT-scale baryogenesis will not be erased by electroweak interactions. In the Minimal Standard Model if $B - L$ is non-zero, then the equilibrium baryon number at high temperature is $B \sim (B - L)$ [63, 66]. However, in minimal GUTs $B - L = 0$, thus again some extensions of standard models are necessary.

There are also models considering cases when the early generated BAU will not be washed out even if $B - L = 0$. For example, in [67] it was argued, that equilibrium anomalous electroweak B -non-conserving interactions do not wash all the baryon asymmetry if (1) there is no mixing in the leptonic sector, (2) there is large flavor asymmetry in the leptonic decays, (3) mass of the Higgs boson is larger than 56 GeV. The idea of asymmetry in leptonic sector was used also in [65] where it was claimed that different lepton masses provide the different leptonic chemical potentials and a net B -asymmetry survives. In the work [68] the conclusion was made that if there is another global quantum number in the low-energy theory that is, like B and L , conserved up to weak anomaly, the electroweak anomaly processes would not necessarily erase a primordial baryon asymmetry even if $B - L = 0$. There are also considerations of SUSY effects, which can save primordial BAU [69].

In any case the above said shows that the question of GUT-scale baryogenesis is still open and needs further investigation.

4.2 Particle-antiparticle mass-difference and Baryogenesis in $SU(5)$ model

As we have discussed in previous sections, the baryon excess is usually expected to be produced during the nonequilibrium stage of the Universe expansion, since if CPT is valid, no non-conserved quantum number (such as baryon number) could take a nonzero value when averaged over a statistical at thermodynamic equilibrium. Now we shall take into account that CPT is not valid due to Universe expansion and thus masses of particles and antiparticles are slightly different. One might expect that there could exist some excess in the number density of particles over antiparticles (or vice-versa) in plasma even at thermal equilibrium. This could in turn result in the non-vanishing average macroscopic value of the baryon number in the plasma at equilibrium. We find that this is indeed the case. Let us consider this in more detail.

Let us discuss, what does happen in cosmological plasma in thermal equilibrium at high temperatures in the frames of some GUT theory if the masses of leptoquarks and antileptoquarks are different. The thermal equilibrium in an expanding Universe establishes when the particle interaction rates are rapid compared to the expansion rate. We shall consider the temperature interval $10^{15} GeV \sim M_{X,Y} > T \geq T_f \sim 10^{14} GeV$, where $M_{X,Y}$ is the mass of leptoquarks and $T_f \sim 0.1 M_X$ is the "freezing" temperature at which the massive leptoquarks in plasma decouple.

As a starting point we will assume that the World is described by the minimal $SU(5)$ model. The particle contents of this model is as follows (see e.g. [41]):

(a). Three families of quarks and leptons. Each family (labelled by (j)) contains 15 twocomponent fermions:

$$(\nu, e^-)_{(j)L} : (\mathbf{1}, \mathbf{2}); \quad e_{(j)L}^+ : (\mathbf{1}, \mathbf{1}); \quad (u_\alpha, d_\alpha)_{(j)L} : (\mathbf{3}, \mathbf{2});$$

$$u_{(j)L}^{c\alpha} : (\mathbf{3}^*, \mathbf{1}); \quad d_{(j)L}^{c\alpha} : (\mathbf{3}^*, \mathbf{1}).$$

Here the index " L " denotes that fermions are left-handed, the index $\alpha = 1, 2, 3$ corresponds to different colors of particles, the upper index " c " denotes the charge conjugation ($\psi^c = C\gamma^0\psi^* = i\gamma^2\psi^*$, $\psi_L^c = C\bar{\psi}_R^T$), the entries in brackets (n_3, n_2) represent the representations under the $SU(3)^{col}$ and $SU(2)$ subgroups. These fermions can be placed in $\mathbf{5}^* + \mathbf{10}$ representation of $SU(5)$.

(b). The 24 gauge bosons: one photon $(\mathbf{1}, \mathbf{1})$, 8 gluons $G_\beta^\alpha : (\mathbf{8}, \mathbf{1})$, three weak bosons $W^\pm, W^0 : (\mathbf{1}, \mathbf{3})$, six (X, Y) bosons $: (\mathbf{3}, \mathbf{2}^*)$ and their six antiparticles $(\bar{X}, \bar{Y}) : (\mathbf{3}^*, \mathbf{2})$. X and Y bosons have electric charge equal to $4/3$ and $1/3$ respectively.

(c). The 34 scalar bosons of the model are placed in $\mathbf{24}$ and complex $\mathbf{5}$ representations of $SU(5)$. Out of them the physical meaning have: the complex Higgs doublet of Weinberg-Salam model $((H_d^+, H_d^0) : (\mathbf{1}, \mathbf{2})$ and (H_d^0, H_d^-)); the complex color triplet of Higgs fields $H_t : (\mathbf{3}, \mathbf{1})$; the superheavy scalar bosons placed in representation $\mathbf{24}$: $\Phi_\beta^\alpha : (\mathbf{8}, \mathbf{1})$; $\Phi_s^r : (\mathbf{1}, \mathbf{3})$ and $\Phi : (\mathbf{1}, \mathbf{1})$. The remaining twelve bosons from representation $\mathbf{24}$ are unphysical and give masses to X and Y gauge bosons. Here the indices r, s run through the values 4, 5.

In $SU(5)$ there are two types of bosons that mediate B - and L -violating interactions — the X and Y gauge particles and color triplet of Higgs scalars H_t . However H bosons typically have a smaller effective coupling constant ($\alpha_H \sim 10^{-4} \div 10^{-6}$) than X, Y bosons do ($\alpha \sim 10^{-2}$ (see e.g. [39]) and here we will only consider the effect of the X and Y gauge bosons on the evolution of Baryon asymmetry.

The super-heavy scalars Φ from representation $\mathbf{24}$ do not couple to fermions and we will not consider them too. Neglecting interactions with heavy H_t and Φ scalars does not affect our calculations sufficiently.

Thus for our purposes the relevant fields are: three generations of quarks and leptons, complex doublet of Higgs particles and gauge particles. At the temperature interval which we are considering all particles except X and Y can be treated as a massless with a very good approximation.

Since $SU(5)$ is symmetrical with respect to generations and colors, we shall treat generation and color as additional, spinlike degeneracies (recall that the number of spin states for the particle species i are equal to $2s_i + 1$ for $m_i > 0$; to 2 for $m_i = 0, s_i > 0$ and to 1 for $m_i = 0, s_i = 0$, where s_i denotes the spin of " i " particle and m_i — its mass). Specifically, all left- and right-handed up-like quarks ($u_{(j)L}$ and $u_{(j)R}$) will be referred as u_L and u_R with an effective degeneracy factor $g_{u_L} = g_{u_R} = g_u = (1 \text{ helicity state}) \times (3 \text{ generations}) \times (3 \text{ colors}) = 9$; all left- and right-handed down-like quarks ($d_{(j)L}$ and $d_{(j)R}$) are designated d_L and d_R with an effective degeneracy $g_{d_L} = g_{d_R} = g_d = 9$. All left- and right-handed charged leptons ($e_{(j)L}$ and $e_{(j)R}$) are labeled by e_L and e_R and have an effective degeneracy $g_{e_L} = g_{e_R} = g_e = (1 \text{ helicity state}) \times (3 \text{ generations}) = 3$ and finally, all neutrinos $\nu_{(j)}$ are labeled by ν , where ν has an effective degeneracy $g_\nu = (1 \text{ helicity state}) \times (3 \text{ generations}) = 3$.

The X and Y superheavy gauge bosons have an effective degeneracy $g_X = g_Y = (3 \text{ spin states}) \times (3 \text{ colors}) = 9$. The X and Y masses are assumed to be equal and each

is denoted M_X . Any difference is due to $SU(2) \times U(1)$ weak symmetry breaking and is of order 100GeV, negligible at temperatures which we are investigating.

Since we are investigating a cosmological plasma in thermal equilibrium, the particle distributions will be given by their thermal forms. If we consider a uniform ideal gas of particle species " i " with mass m_i in thermal equilibrium at temperature T , then the number density of such particles in phase space is given by (see [48])

$$\frac{dN_i}{d^3\vec{p}d^3x} \equiv f_i(p) = \frac{g_i}{(2\pi)^3} \cdot \frac{1}{e^{(E_i - \mu_i)/T} + \theta}. \quad (4.3)$$

Here $E_i = \sqrt{\vec{p}^2 + m_i^2}$, $\theta = \pm 1$ for particles obeying Fermi-Dirac (Bose-Einstein) statistics and $\theta = 0$ in the classical (distinguishable particles) approximation of Maxwell-Boltzmann statistics; g_i gives the effective degeneracy for particle species " i " (see above). The μ is a possible chemical potential which serves to constrain the total number of particles.

The total particle number density may be obtained by integrating (4.3) over available momentum states:

$$\begin{aligned} n_i(T, \frac{m_i}{T}, \frac{\mu_i}{t}) &= \int_0^\infty \frac{d^3\vec{p}}{2\pi^3} \cdot \frac{g_i}{e^{(E_i - \mu_i)/T} + \theta} = \\ &= \frac{g_i T^3}{2\pi^3} \cdot \int_{m_i/T}^\infty q \sqrt{q^2 - (m_i/T)^2} \cdot [e^{q - \mu_i/T} + \theta]^{-1} dq. \end{aligned}$$

We shall make the simplifying approximation that all particles obey Maxwell-Boltzmann statistics. The corrections resulting the indistinguishability of the particles are small (see [48]). In this approximation the number density integral becomes:

$$n_i(T, \frac{m_i}{T}, \frac{\mu_i}{t}) \simeq g_i \cdot \left(\frac{m_i T}{2\pi}\right)^{3/2} \cdot e^{-(m_i - \mu_i)/T} \cdot \left[1 + \frac{15}{8} \frac{T}{m_i} + \dots\right] \quad (for T \ll m_i); \quad (4.4)$$

$$n_i(T, \frac{m_i}{T}, \frac{\mu_i}{t}) \simeq g_i \cdot \left(\frac{T^3}{\pi^2}\right) \cdot e^{\mu_i/T} \cdot \left[1 - \frac{1}{4} \left(\frac{m_i}{T}\right)^2 - \dots\right] \quad (for T \gg m_i); \quad (4.5)$$

It is easy to see from (4.4) and (4.5) that the excess of particle species " i " over their antiparticles can be determined via chemical potentials. Thus now we have to discuss the relevant chemical potentials of used model.

In thermal equilibrium any charged particle can emit or absorb an arbitrary number of photons; thus the chemical potential of the photon vanishes at sufficiently high temperatures and densities. An analogous conclusion holds for the Z^0 boson (for $T \geq M_{Z^0}$). Therefore, as a consequence of processes such as $\gamma\gamma \rightarrow b\bar{b}$, the chemical potentials of particles and antiparticles are equal and opposite. At temperatures above the electroweak phase transition (when $U(2)$ is an exact symmetry) the W^\pm also have a vanishing chemical potentials, what imposes equality of the chemical potentials for fields in the same electroweak multiplet (so that

$\mu(X) = \mu(Y)$, $\mu(u_L) = \mu(d_L)$, $\mu(e_L) = \mu(\nu)$). Similarly (as $SU(3)$ is an exact symmetry) the chemical potential of gluon vanishes and thus the different colored quarks have equal chemical potentials. Moreover, Cabbibo mixing will guarantee that chemical potentials of all up-like quarks and all down-like quarks are respectively equal ($\mu(u_{(j)}) = \mu(u)$; $\mu(d_{(j)}) = \mu(d)$). In the absence of flavor mixing neutrino processes (e.g. due to neutrino masses), the lepton generations will not in general have an equal chemical potentials, but as we shall see soon, interactions with leptoquarks and quarks ensure that leptons of all generations have equal chemical potentials. However, the chemical potentials of left particles, in general, do not equal to those of right particles. In all, there are 7 different chemical potentials which we have to calculate. Fortunately thermodynamical equilibrium imposes a number of relations between them.

Recall, that whenever interactions are in thermal equilibrium, the sum of the chemical potentials of the incoming particles is equal to that of outgoing particles (see [70]). Rapid electroweak interactions in the early Universe enforce the following equilibrium relations among the chemical potentials (see [66, 65] and [39, 41, 49]):

$$\begin{aligned}
(a). \mu(u_R) &= -\mu(u_L^c) = -\mu(u_L) + \mu(X) & (u_L \leftrightarrow u_L^c + X); \\
(b). \mu(H_d^0) &= -\mu(u_L) + \mu(u_R) = -2\mu(u_L) + \mu(X) & (H_d^0 \leftrightarrow \bar{u}_L + u_R); \\
(c). \mu(d_R) &= -\mu(H_d^0) + \mu(u_L) = 3\mu(u_L) - \mu(X) & (H_d^0 \leftrightarrow d_L + \bar{d}_R); \\
(d). \mu(e_{(j)R}) &= -\mu(e_{(j)L}^+) = -\mu(u_L) - \mu(X) & (X + d_L \leftrightarrow e_{(j)L}^+); \\
(e). \mu(e_{(j)L}) &= -\mu(H_d^0) + \mu(e_{(j)R}) = -3\mu(u_L) & (H_d^0 \leftrightarrow e_{(j)L}^+ \bar{e}_{(j)R}). \tag{4.6}
\end{aligned}$$

The other reactions are correlated with these by $SU(2)$ symmetry.

We see, that by the use of these relations all of the chemical potentials can be expressed by means of $\mu(u_L)$ and $\mu(X)$.

We have also one more unknown variable $\Delta m/m$.

To evaluate this remaining quantities one can use the restrictions coming from conservation of electric charge, color, weak isospin and $B - L$. I.e. the cosmological plasma must be neutral with respect to charge, color, weak isospin and $B - L$. The neutrallness with respect to color and weak isospin is satisfied automatically, since at the temperature interval, which we are considering, $SU(3)^{col}$ and $SU(2)$ are exact symmetries. The conditions of charge and $B - L$ neutrallness can be expressed in terms of particle densities in plasma and have the form:

$$\begin{aligned}
Q &= \frac{4}{3} [n(X) - n(\bar{X})] - \frac{1}{3} [n(Y) - n(\bar{Y})] + \frac{2}{3} [n(u) - n(\bar{u})] - \\
&\frac{1}{3} [n(d) - n(\bar{d})] - [n(e) - n(e^+)] - [n(W^-) - n(W^+)] + [n(H_d^+) - n(H_d^-)] = 0 \tag{4.7}
\end{aligned}$$

$$\begin{aligned}
B - L &= \frac{2}{3} [n(X) - n(\bar{X})] + \frac{2}{3} [n(Y) - n(\bar{Y})] + \frac{1}{3} [n(u) - n(\bar{u})] + \\
&\frac{1}{3} [n(d) - n(\bar{d})] - [n(e) - n(e^+)] - [n(\nu) - n(\bar{\nu})] = 0 \tag{4.8}
\end{aligned}$$

From (4.4) and (4.5) it is easy to see, that the difference between the densities of given particle species and the densities of their antiparticles can be expressed via their chemical potentials.

We shall assume that $X(Y)$ and $\bar{X}(\bar{Y})$ have the different masses

$$\begin{aligned} M_{X(Y)} &= m + \Delta m; \\ M_{\bar{X}(\bar{Y})} &= m - \Delta m. \end{aligned}$$

and use for their densities formula (4.4). Then, assuming that $\delta m/m \ll 1$, $\mu/T \ll 1$, $\Delta m/T \sim 0.1 \ll 1$, we yield (hereafter, if not necessary, we shall write simply X instead of X, Y):

$$\begin{aligned} n(X) - n(\bar{X}) &\simeq g_X \cdot \left(2 \frac{T^3}{\pi^2}\right) \cdot \left(\frac{\sqrt{\pi}}{2\sqrt{2}}\right) \cdot \left[\left(\frac{m}{T}\right)^{3/2} \cdot e^{-m/T}\right] \cdot \left(\frac{\mu(X)}{T} - \frac{\Delta m}{T} - \frac{3}{8} \frac{\Delta m}{m}\right) \approx \\ &\approx g_X \cdot \left(2 \frac{T^3}{\pi^2}\right) \cdot 8.9 \cdot 10^{-4} \cdot \left(\frac{\mu(X)}{T} - \frac{\Delta m}{T}\right) \end{aligned} \quad (4.9)$$

For all the other particles we must use formula (4.5) since their masses are zero or much less than T . Then we yield:

$$n(i) - n(\bar{i}) = g_i \cdot \left(2 \frac{T^3}{\pi^2}\right) \cdot \frac{\mu(i)}{T} \quad (4.10)$$

Substituting this formulae into plasma neutrallness conditions (4.7) and (4.8) we yield two additional equations establishing correlations between the chemical potentials and $\Delta m/m$, which after using the relations (4.6) take the form (we have divided them by the factor $2T^3/\pi^2$):

$$\begin{aligned} Q &= -5 \cdot (2.67 \cdot 10^{-3}) \cdot \frac{\Delta m}{T} - 2 \cdot \frac{\mu(u_L)}{T} + 13 \cdot \frac{\mu(X)}{T} = 0 \\ B - L &= -4 \cdot (2.67 \cdot 10^{-3}) \cdot \frac{\Delta m}{T} + 33 \cdot \frac{\mu(u_L)}{T} + 3 \cdot \frac{\mu(X)}{T} = 0 \end{aligned}$$

Solving this system of equations gives

$$\begin{aligned} \mu(u_L) &\simeq 0.21\mu(X) \\ \mu(X) &\simeq 1.07 \cdot 10^{-3} \Delta m. \end{aligned} \quad (4.11)$$

Now we can express the particle-over-antiparticle excess for leptoquarks and quarks via the mass difference Δm . This is necessary to find the net baryon number carried by leptoquarks and quarks. Using (4.9), (4.10) and (4.11) we yield:

$$\frac{n(X) + n(Y) - n(\bar{X}) - n(\bar{Y})}{n_\gamma} \simeq (g_X + g_Y) \cdot 8.9 \cdot 10^{-4} \cdot \left(\frac{\mu(x)}{T} - \frac{\Delta m}{T}\right) \simeq -1.6 \cdot 10^{-1} \cdot \frac{\Delta m}{m} \quad (4.12)$$

$$\frac{n(q) - n(\bar{q})}{n_\gamma} \simeq g_{u_L} \cdot \frac{\mu(u_L)}{T} + g_{u_R} \cdot \frac{\mu(u_R)}{T} + g_{d_L} \cdot \frac{\mu(d_L)}{T} + g_{d_R} \cdot \frac{\mu(d_R)}{T} \simeq 0.8 \cdot 10^{-1} \cdot \frac{\Delta m}{m} \quad (4.13)$$

Thus for the baryon number excess we have

$$\frac{\Delta B_{X,Y}}{n(\gamma)} = \frac{1}{6} \frac{\Delta n(X)}{n(\gamma)} \simeq -0.25 \cdot 10^{-1} \cdot \frac{\Delta m}{m}$$

$$\frac{\Delta B_q}{n(\gamma)} = \frac{1}{n(\gamma)} (\Delta n(u) + \Delta n(d)) \simeq 0.8 \cdot 10^{-1} \cdot \frac{\Delta m}{m}$$

Here $\Delta B_{X,Y}$ and ΔB_q is the net baryon number carried by leptiquarks and quarks respectively; the factor $1/6$ in (4.12) is the mean excess of baryon number in decays of single leptiquark in vacuum (see [39]); for $n(\gamma)$ we have used formula (4.5) with $m_\gamma = 0, \mu(\gamma) = 0$.

Using (4.12) and (4.13) one can see that the complete BAU is

$$\delta \simeq \frac{1}{g_*} \left(\frac{\Delta B_{X,Y}}{n_\gamma} + \frac{\Delta B_q}{n_\gamma} \right) \simeq 0.55 \cdot 10^{-3} \frac{\Delta m}{m}. \quad (4.14)$$

Thus we have found that if there exists the mass difference between leptiquarks and antileptiquarks then in primordial plasma there will be the excess of baryons over antibaryons even in thermal equilibrium. Comparing this result with observed data one can conclude that under the assumption that BAU is completely caused by *CPT*-violation this mass difference must be of the order of

$$\frac{\Delta m}{m} \sim 10^{-4} \div 10^{-6}.$$

4.3 Calculation of BAU caused by the Universe expansion

As we have already discussed, the standard $SU(5)$ GUT fails to explain the observable BAU since the lack of sufficient nonequilibrium in this model. In previous section it was shown that if one takes into account that *CPT* theorem is violated on GUT scale and causes the mass difference between the leptiquarks and antileptiquarks

$$\Delta m \sim (10^{-4} \div 10^{-6})m, \quad (4.15)$$

one can obtain the observable value of BAU without the requirement of nonequilibrium.

Now let us show that *CPT*-invariance in FRW space is really violated and on GUT scale it causes the mass difference (4.15).

Our task is to find the quantity $\Delta m_X/m_X$. We have already obtained that in RW universe the mass difference obtained between particles and their antiparticles via Higgs mechanism is

$$\frac{\Delta m_X}{m_X} = (\dot{a}|_{\tau=0})\tau/a_0. \quad (4.16)$$

Let us consider the case of leptiquarks. As the Universe cools down the first phase transition takes place at the GUT scale at the time moment

$$t \sim \frac{1}{M_X}.$$

At this moment the group of GUT, for example $SU(5)$, breaks down and the leptoquarks acquire their masses. All the other particles of the model are massless at this scale. Let us assume that the Leptoquarks acquire their masses during a sufficiently short period

$$t \sim a_0 \tau \sim (1 \div 10^{-1})/m_X, \quad \tau \ll 1.$$

Then we can use the short-time-interval approximation and so the expression (4.16) is valid.

Let us evaluate this value for the different expansion regimes [25] .

As we have seen, during the period when $\rho = \text{const}$ the evolution of the Universe is described (according to (2.8)) by the de Sitter expansion law

$$a = a_0 \cdot \exp(Ht) = a_0/(1 - a_0 H \tau) \quad (4.17)$$

with the Hubble constant

$$H = K\sqrt{\rho},$$

where the constant K is expressed by (2.7)

The energy density ρ of a nonlinear scalar field leading to inflation in some models is of the order of

$$\rho \sim (10^{-2} \div 1)m_X^4.$$

From (4.17) it follows that

$$\dot{a}|_{\tau=0} = a_0^2 H$$

and according to (4.16) we obtain:

$$\frac{\Delta m_X}{m_X} \sim H a_0 \tau \sim H t \sim (10^{-2} \div 1) \cdot \frac{m_X}{M_{pl}}. \quad (4.18)$$

If the mass of the leptoquarks is generated at the very end of the inflationary period, then, taking into account that $m_X/M_{pl} \sim 10^{-4}$, eq. (4.18) gives the value (4.15), required to explain the BAU.

When the ultrarelativistic gas of noninteracting particles dominates in the Universe ($\rho = a^{-4}$), then according to (2.9) the scale factor depends on time through the power-function law:

$$a = (a_0^2 + 2Kt)^{1/2} = a_0 + K\tau.$$

Then

$$a|_{\tau=0} = K$$

and

$$\frac{\Delta m_X}{m_X} \sim \frac{K}{a_0} \tau \sim \frac{K}{a_0^2} t. \quad (4.19)$$

If leptoquarks acquire their mass during this period and the value of scale factor at the initial moment of phase transition is of the order of

$$a_0 \sim (1 \div 10)/m_X,$$

then we obtain (4.15) again. Note that at the power-function-like expansion of the Universe the mass difference between particles and antiparticles is proportional to a_0^{-2} (see (4.19)) and for the particles acquiring their mass later (at large a_0) it tends to

zero. This result is valid also for the expansion law $a \sim t^{2/3}$. Thus the CPT -violation causing the mass difference between the particles and antiparticles is essential only during the early stages of the universe expansion.

We have shown that for a wide spectrum of models the observed BAU can be explained by the violation of the CPT -theorem in the strong gravitational fields at the early stages of the Universe expansion without requirement of nonequilibrium and additional complication of model. Besides, as we have seen in subsection 3.3, in the expanding universe Δm_X is positive and according to (4.14) we have the predominance of matter over antimatter. In the collapsing universe Δm_X is negative and we should observe the opposite picture. Thus the predominance of matter over antimatter and the expansion of the Universe seem to be connected facts. This conclusion is valid also with respect to the other models of GUT (besides of $SU(5)$) though the numerical evaluations can be different.

At the end we would like to note that to discern the properties of particles and antiparticles in Minkowski space it is necessary to have the C -noninvariant interaction. However in the curved space it is not so if CPT is violated. As we have shown, the different behavior of particles and antiparticles can be caused by a cosmological T -asymmetry and thus BAU can be generated without C -violation.

Bibliography

- [1] P.C.W.Davies. *The Physics of Time Asymmetry*. (Surrey Univ.Press, 1974); *Space and Time in the Modern Universe*. (Cambridge Univ.Press, Cambridge 1977).
- [2] A.Grunbaum. *Philosophical problems of Space and Time*. (Knopf, NY, 1963) .
- [3] H.Reichenbach. *The direction of time*. (Univ. of California Press, Berkeley, 1956).
- [4] S.W.Hawking. *A Brief History of Time*. (Bantam Books, Toronto, 1988).
- [5] R.Penrose. in: *General Relativity*. eds. S.W.Hawking and W.Israel (Cambridge Univ.Press. Cambridge, 1979).
- [6] W.M.Gibson, B.R.Pollard. *Symmetry Principles in Elementary Particle Physics*. (Cambridge Univ.Press, Cambridge, 1976).
- [7] T.D.Lee. *Particle Physics and Introduction to Field Theory. Vol.1* ed. H.Primakoff (Harwood Academic Publishers, NY, 1981).
- [8] S.W.Hawking. **Phys.Rev. D13** (1976) 191; *ibid.* **D14** (1976) 2460.
- [9] J.D.Barrow, J.D.Tipler. *The Anthropic Cosmological Principle* (Clarendon Press, Oxford, 1986).
- [10] G.Steigman. **Ann.Rev.Astron.Astrophys. 14** (1976) 339.
- [11] E.W.Kolb, M.S.Turner. *The Early Universe* (Addison-Wesley, Redwood City, Calif., 1989).
- [12] G.Börner. *The Early Universe* (Springer, Berlin, 1986).
- [13] N.A.Chernikov. in: *Proc.Int.Conf. on Non-Local Quantum Field theory*. (Dubna, 1967) p.183.
- [14] K.A.Bronnikov, E.A.Tagirov. Preprint P2-4151, JINR, 1968.
- [15] L.Parker. **Phys.Rev.Lett. 21** (1968) 562; **Phys.Rev. 183** (1969) 1057; in: *Proc. of the Symposium on Asymptotic Properties of Space-Time*. ed. F.P.Esposito and L.Witten (Plenum Publ.Corp., NY, 1977) p.107.
- [16] A.A.Grib, S.G.Mamayev **Yad.Fiz. 10** (1969) 1276.

- [17] N.D.Birrell and L.H.Ford. **Ann. of Phys (NY).** **122** (1979) 1;
 N.D.Birrell, P.C.W.Davies and L.H.Ford. **J.Phys. A** **13** (1980) 961;
 N.D.Birrell. in: *Quantum Gravity 2 — A Second Oxford Symposium*. eds.
 C.J.Isham, R.Penrose and D.W.Sciama (Clarendon Press, Oxford, 1981) p. 164;
 L.H.Ford. in: *Quantum Theory of Gravity — Essays in Honour of the 60-th
 Birthday of Bryce De Witt*. ed. S.M.Christensen (Adam Hilger, Bristol, 1984) p.
 125.
- [18] R.Brout and F.Englert. **Nucl.Phys. B180 [FS2]** (1981) 181.
- [19] S.Barshay. **Phys.Lett. 101B** (1981) 155.
- [20] L.H.Ford. **Nucl.Phys. B204** (1982) 35.
- [21] V.A.Kuzmin. in: *Seminar Proc. Quarks-84. Vol. 1*. eds. A.N.Tavkhelidze et al.
 (INR Press, Moscow, 1985) [in Russian]; in: *Proc.III Seminar on Quantum
 Gravity*. (Moscow, 1984); Preprint NORDITA-85/4 (1985).
- [22] K.-H.Lotze. **Class. Quantum Grav. 2** (1985) 351, 363;
 K.-H.Lotze. **Class. Quantum Grav. 3** (1986) 81.
- [23] K.-H.Lotze. in: *Proc. of the XX Int. Symposium on the Theory of Elementary
 Particles. Ahrenshoop - 1986*. ed. F.Kaschluhn (Berlin - Zeuthen - DDR, 1986)
 p. 226.
- [24] A.A.Grib, Yu.V.Kryukov. **Yad.Fiz. 48** (1988) 1842
- [25] A.Barnaveli, M.Gogberashvili. **Phys.Lett. B 316** (1993) 57; **Yad.Fiz. 57**
 (1994) 958.
- [26] A.A.Grib, S.G.Mamayev, V.M.Mostepanenko. *Quantum Effects in Strong Exter-
 nal Fields*. (Atomizdat, Moscow, 1980). [in Russian].
- [27] N.D.Birrell, P.C.W.Davies. *Quantum Fields in the Curved Space-time*. (Cam-
 bridge Univ. Press. Cambridge 1982).
- [28] R.C.Tolman *Relativity, Thermodynamics and Cosmology*. (Clarendon Press, Ox-
 ford, 1969).
- [29] U.Canuto et. al. **Phys.Rev. D16** (1977) 1643.
- [30] A.A.Grib. **Vestnik LGU, 22** (1967) 50 [in russian].
- [31] S.W.Hawking. **Phys.Rev. D14** (1975) 2460; **Comm.Math.Phys. 87** (1982)
 395.
- [32] R.M.Wald *General Relativity*. (Univ. of Chicago Press, Chicago, 1984).
- [33] T.Banks, M.E.Peskind, L.Susskind. **Nucl.Phys. B244** (1984) 125.
- [34] G.Lüders. **Dan.Mat.Fys.Medd. 28** (1954) 5; **Ann.Phys. 2** (1957) 1 ;
 J.Schwinger. **Phys.Rev. 82** (1951) 914.

- [35] W.Pauli. in: *Niels Bohr and the Development of Physics* ed. W.Pauli (Pergamon Press, NY, 1955).
- [36] S.L.Meyer et. al. **Phys.Rev.** **132** (1963) 2693.
- [37] Particle Data Group. **Phys.Rev.** **D45** (1992) 1.
- [38] M.Kobayashi, A.I.Sanda **Phys.Rev.Lett.** **69** (1992) 3139;
M.Hayakawa, A.I.Sanda. **Phys.Rev.** **D48** (1993) 1150.
- [39] J.N.Fry, K.A.Olive, M.S.Turner. **Phys.Rev.** **D22** (1980) 2953, 2977.;
Phys.Rev.Lett. **45** (1980) 2074.
- [40] A.D.Dolgov, Ya.B.Zeldovich. **Uspekhi.Fiz.Nauk.** **130** (1980) 559.
- [41] P.Langaker. **Phys.Rep.** **72** (1981) 185 .
- [42] J.D.Barrow, E.J.Copeland, E.W.Kolb, A.R.Liddle. **Phys.Rev.** **D43** (1991) 977, 984.
- [43] J.D.Barrow. **Surveys of H.E.Phys.** **1** (1980) 183.
- [44] Ya.B.Zeldovich, I.D.Novikov. *Structure and Evolution of the Universe* (Nauka, Moscow, 1975)
- [45] E.W.Kolb. in: *CP Violation in Particle Physics and Astrophysics* ed. J. Tran Thanh Van (Editions Frontieres, France, 1989) p.605.
- [46] A.D.Sakharov. **JETP Lett.** **5** (1967) 24.
- [47] V.A.Kuzmin. **JETP Lett.** **12** (1970) 228.
- [48] E.W.Kolb, S.W.Wolfram. Preprint OAP-579, CALT-68-754 (1979); **Nucl.Phys.** **B172** (1980) 224 (E: **B195** (1982) 542.
- [49] D.W.Nanopoulos, S.Weinberg. **Phys.Rev.** **D20** (1979) 2484.
- [50] R.D.Peccei. in: *CP Violation in Particle Physics and Astrophysics* ed. J. Tran Thanh Van (Editions Frontieres, France, 1989) p.615.
- [51] S.Barr, G.Segre, H.A.Weldon. **Phys.Rev.** **D20** (1979) 2494.
- [52] S.Dimopoulos, L.Susskind. **Phys.Rev.** **D18** (1976) 4500.
- [53] S.Weinberg. **Phys.Rev.Lett.** **42** (1979) 85.
- [54] D.Toussaint, S.B.Treiman, F.Wilchek, A.Zee. **Phys.Rev.** **D19** (1979) 1036.
- [55] V.A.Kuzmin, M.E.Shaposhnikov, I.I.Tkachev. **Nucl.Phys.** **B196** (1982) 29.
- [56] G.t'Hooft. **Phys.Rev.** **D14** (1976) 3432.
- [57] N.S.Manton. **Phys.Rev.** **D28** (1983) 2019;
F.R.Klinkhammer, N.S.Manton. **Phys.Rev.** **D30** (1984) 2212.

- [58] V.A.Kuzmin, V.A.Rubakov, M.E.Shaposhnikov. **Phys.Lett** **155B** (1985) 36.
- [59] V.A.Matveev, V.A.Rubakov, A.N.Tavkhelidze, M.E.Shaposhnikov **Uspekhi Fiz. Nauk** **156** (1988) 193.
- [60] G.R.Farrar, M.E.Shaposhnikov. Preprint CERN-TH. 6734/93, RU-93-11 (1993).
- [61] M.B.Gavela, P.Hernandez, J.Orloff, O.Pene. Preprint CERN-TH.7081/93, LP THE Orsay-93/48, HUTP-93/48, HD-THEP-93-45 (1993);
P.Huet, E.Sather. Preprint SLAC-PUB-6479, May, (1994).
- [62] A.Cohen, D.Kaplan, A.Nelson. Preprint UCSD-PTH-93-02, BUHEP-93-4 (1993)
- [63] A.Dolgov. **Phys.Rep.** **222** (1992) 309.
- [64] A.Barnaveli, M.Gogberashvili. In preparation (1994)
- [65] H.Dreiner, G.G.Ross. Preprint OUTP-92-08p (1992).
- [66] J.A.Harvey, M.S.Turner. **Phys.Rev.** **D42** (1990) 3344.
- [67] V.A.Kuzmin, V.A.Rubakov, M.E.Shaposhnikov. Preprint NBI-HE-87-05 (1987)
- [68] S.M.Barr. **Phys.Rev.** **D44** (1991) 3062.
- [69] L.E.Ibañez, F.Quevedo. Preprint CERN-TH-6433/92 (1992)
- [70] L.D.Landau and E.M.Lifshitz. *Statistical Physics, Part 1, 3rd Edition* (Pergamon Press 1982).